SECTION 6

FLUID MECHANICS, PUMPS, PIPING, AND HYDRO POWER

Part 1 Fluid Mechanics

Part 2 Pump Operating Modes, Affinity Laws, Speed, and Head

Part 3 Centrifugal Pumps and Hydro Power

PART 1

FLUID MECHANICS

Hydrostatics

The notational system used in hydrostatics is as follows: W = weight of floating body, lb (N); V = volume of displaced liquid, ft^3 (m^3); w = specific weight of liquid, lb/ft^3 (N/m^3); for water w = 62.4 lb/ft^3 (9802 N/m^3), unless another value is specified.

BUOYANCY AND FLOTATION

A timber member 12 ft (3.65 m) long with a cross-sectional area of 90 sq.in. (580.7 cm^2) will be used as a buoy in saltwater. What volume of concrete must be fastened to one end so that 2 ft (60.96 cm) of the member will be above the surface? Use these specific weights: timber = 38 lb/ft^3 (5969 N/m^3); saltwater = 64 lb/ft^3 (10,053 N/m^3); concrete = 145 lb/ft^3 (22,777 N/m^3).

Calculation Procedure:

1. *Express the weight of the body and the volume of the displaced liquid in terms of the volume of concrete required*

Archimedes' principle states that a body immersed in a liquid is subjected to a vertical buoyant force equal to the weight of the displaced liquid. In accordance with the equations of equilibrium, the buoyant force on a floating body equals the weight of the body. Therefore,

$$W = Vw \tag{1}$$

Let x denote the volume of concrete. Then $W = (90/144)(12)(38) + 145x = 285 + 145x$; $V = (90/144)(12 - 2) + x = 6.25 + x$.

2. *Substitute in Eq. 1 and solve for x*

Thus, $285 + 145x = (6.25 + x)64$; $x = 1.42$ ft^3 (0.0402 m^3).

HYDROSTATIC FORCE ON A PLANE SURFACE

In Fig. 1, *AB* is the side of a vessel containing water, and *CDE* is a gate located in this plane. Find the magnitude and location of the resultant thrust of the water on the gate when the liquid surface is 2 ft (60.96 cm) above the apex.

Calculation Procedure:

1. *State the equations for the resultant magnitude and position*

In Fig. 1, *FH* denotes the centroidal axis of area *CDE* that is parallel to the liquid surface, and *G* denotes the point of application of the resultant force. Point *G* is termed the *pressure center.*

Let A = area of given surface, sq.ft. (cm^2); P = hydrostatic force on given surface, lb (N); p_m = mean pressure on surface, lb/sq.ft. (kPa); y_{CA} and y_{PC} = vertical distance from centroidal axis and pressure center, respectively, to liquid surface, ft (m); z_{CA} and z_{PC} = distance along plane of given surface from the centroidal axis and pressure center, respectively, to line of intersection of this plane and the liquid surface, ft (m); I_{CA} = moment of inertia of area with respect to its centroidal axis, ft^4 (m^4).

Consider an elemental surface of area dA at a vertical distance y below the liquid surface. The hydrostatic force dP on this element is normal to the surface and has the magnitude

$$dP = wy\,dA \tag{2}$$

By applying Eq. 2 develop the following equations for the magnitude and position of the resultant force on the entire surface:

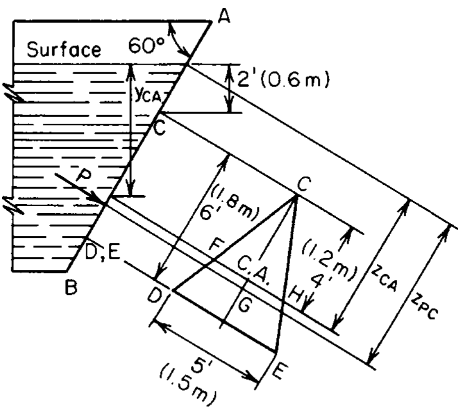


FIGURE 1. Hydrostatic thrust on plane surface.

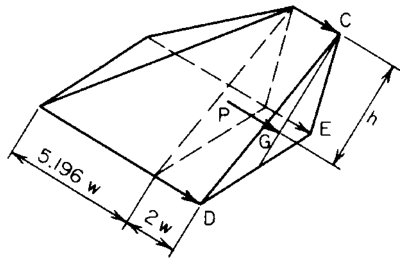


FIGURE 2. Pressure prism.

$$P = wy_{CA}A \tag{3}$$

$$z_{PC} = \frac{I_{CA}}{Az_{CA}} + z_{CA} \tag{4}$$

2. *Compute the required values, and solve the equations in step 1*

Thus $A = \frac{1}{2}(5)(6) = 15$ sq.ft. (1.39 m²); $y_{CA} = 2 + 4 \sin 60° = 5.464$ ft (166.543 cm); $z_{CA} = 2 \csc 60° + 4 = 6.309$ ft (192.3 cm); $I_{CA}/A = (bd^3/36)/(bd/2) = d^2/18 = 2$ sq.ft. (0.186 m²); $P = 62.4(5.464)(15) = 5114$ lb (22.747 N); $z_{PC} = 2/6.309 + 6.309 = 6.626$ ft (201.960 cm); $y_{PC} = 6.626 \sin 60° = 5.738$ ft (174.894 cm). By symmetry, the pressure center lies on the centroidal axis through *C*.

An alternative equation for *P* is

$$P = p_m A \tag{5}$$

Equation 3 shows that the mean pressure occurs at the centroid of the area. The above two steps constitute method 1 for solving this problem. The next three steps constitute method 2.

3. *Now construct the pressure "prism" associated with the area*

In Fig. 2, construct the pressure prism associated with area *CDE*. The pressures are as follows: at apex, $p = 2w$; at base, $p = (2 + 6 \sin 60°)w = 7.196w$.

The force *P* equals the volume of this prism, and its action line lies on the centroidal plane parallel to the base. For convenience, resolve this prism into a triangular prism and rectangular pyramid, as shown.

4. *Determine P by computing the volume of the pressure prism*

Thus, $P = Aw[2 + \frac{2}{3}(5.196)] = Aw(2 + 3.464) = 15(62.4)(5.464) = 5114$ lb (22,747 N).

5. *Find the location of the resultant thrust*

Compute the distance *h* from the top line to the centroidal plane. Then find y_{PC}. Or, $h = [2(\frac{2}{3})(6) + 3.464(\frac{3}{4})(6)]/5.464 = 4.317$ ft (131.582 cm); $y_{PC} = 2 + 4.317 \sin 60° = 5.738$ ft (174.894 cm).

HYDROSTATIC FORCE ON A CURVED SURFACE

The cylinder in Fig. 3*a* rests on an inclined plane and is immersed in liquid up to its top, as shown. Find the hydrostatic force on a 1-ft (30.48-cm) length of cylinder in terms of w and the radius R; locate the pressure center.

Calculation Procedure:

1. *Evaluate the horizontal and vertical component of the force dP on an elemental surface having a central angle dθ*

Refer to Fig. 3*b*. Adopt this sign convention: A horizontal force is positive if directed to the right; a vertical force is positive if directed upward. The first three steps constitute method 1.

Evaluating dP yields $dP_H = wR^2(\sin\theta - \sin\theta\cos\theta)\,d\theta$; $dP_V = wR^2(-\cos\theta + \cos^2\theta)\,d\theta$.

2. *Integrate these equations to obtain the resultant forces P_H and P_V; then find P*

Here, $P_H = wR^2[(-\cos\theta + \frac{1}{2}\cos^2\theta)]_0^{7\pi/6} = wR^2[-(-0.866 - 1) = \frac{1}{2}(0.75 - 1)] = 1.742wR^2$, to right; $P_V = wR^2(-\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta)_0^{7\pi/6} = wR^2(0.5) + (1.833 + 0.217) = 2.550wR^2$, upward; $P = wR^2(1.742^2 + 2.550^2)^{0.5} = 3.087wR^2$.

3. *Determine the value of θ at the pressure center*

Since each elemental force dP passes through the center of the cylinder, the resultant force P also passes through the center. Thus, $\tan(180° - \theta_{PC}) = P_H/P_V = 1.741/2.550$; $\theta_{PC} = 145°41'$.

4. *Evaluate P_H and P_V*

Apply these principles: P_H = force on an imaginary surface obtained by projecting the wetted surface on a vertical plane; $P_V = \pm$ weight of real or imaginary liquid lying between the wetted surface and the liquid surface. Use the plus sign if the *real* liquid lies below the wetted surface and the minus sign if it lies above this surface.

Then P_H = force, to right, on AC + force, to left, on EC = force, to right, on AE; $AE = 1.866R$; $p_m = 0.933\,wR$; $P_H = 0.933\,wR(1.866R) = 1.741\,wR^2$; P_V = weight of imaginary

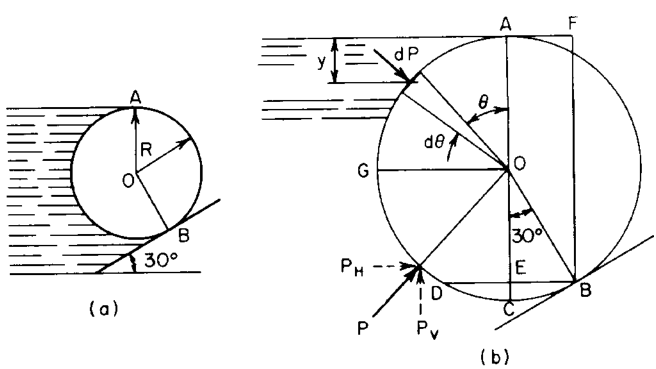


FIGURE 3

liquid above *GCB* − weight of real liquid above *GA* = weight of imaginary liquid in cylindrical sector *AOBG* and in prismoid, *AOBF*. Volume of sector *AOBG* = $[(7\pi/6)/(2\pi)](\pi R^2) = 1.833R^2$; volume of prismoid $AOBF = \frac{1}{2}(0.5R)(R + 1.866R) = 0.717R^2$; $P_V = wR^2(1.833 + 0.717) = 2.550wR^2$.

STABILITY OF A VESSEL

The boat in Fig. 4 is initially floating upright in freshwater. The total weight of the boat and cargo is 182 long tons (1813 kN); the center of gravity lies on the longitudinal (i.e., the fore-and-aft) axis of the boat and 8.6 ft (262.13 cm) above the bottom. A wind causes the boat to list through an angle of 6° while the cargo remains stationary relative to the boat. Compute the righting or upsetting moment (*a*) without applying any set equation; (*b*) by applying the equation for metacentric height.

Calculation Procedure:

1. *Compute the displacement volume and draft when the boat is upright*

The buoyant force passes through the center of gravity of the displaced liquid; this point is termed the *center of buoyancy.* Figure 5 shows the cross section of a boat rotated through an angle ϕ. The center of buoyancy for the upright position is *B*; *B′* is the center of buoyancy for the position shown, and *G* is the center of gravity of the boat and cargo.

In the position indicated in Fig. 5, the weight *W* and buoyant force *R* constitute a couple that tends to restore the boat to its upright position when the disturbing force is removed; their moment is therefore termed *righting.* When these forces constitute a couple that increases the rotation, their moment is said to be *upsetting.* The wedges *OAC* and *OA′C′* are termed the *wedge of emersion* and *wedge of immersion,* respectively. Let h = horizontal displacement of center of buoyancy caused by rotation; h' = horizontal

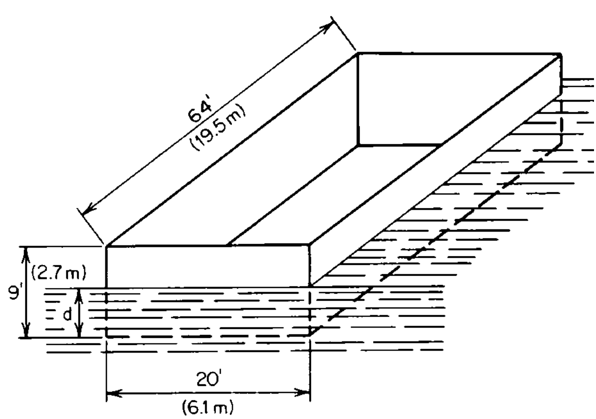


FIGURE 4

distance between centroids of wedge of emersion and wedge of immersion; V' = volume of wedge of emersion (or immersion). Then

$$h = \frac{V'h'}{V} \qquad (6)$$

The displacement volume and the draft when the boat is upright are W = 182 (2240) = 407.700 lb (1813 N); $V = W/w$ = 407.700/62.4 = 6530 ft^3 (184.93 m^3); d = 6530/[64(20)] = 5.10 ft (155.448 cm).

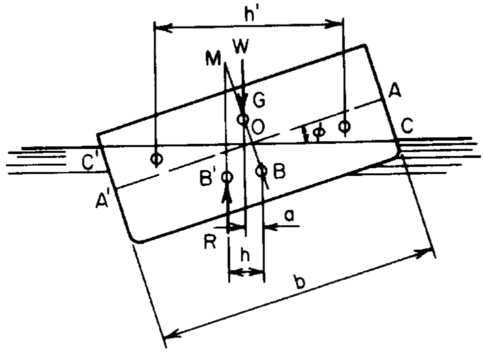


FIGURE 5. Location of resultant forces on inclined vessel.

2. *Find h, using Eq. 6*

Since ϕ is relatively small, apply this approximation: $h' = 2b/3 = 2(20)/3$ = 13.33 ft (406.298 cm), h = $\frac{1}{2}(10)(10 \tan 6°) \times (13.33)/[5.10(20)]$ = 0.687 ft (20.940 cm).

3. *Compute the horizontal distance a (Fig. 5)*

Thus, $BG = 8.6 - \frac{1}{2}(5.10)$ = 6.05 ft (184.404 cm); a = 6.05 sin 6° = 0.632 ft (19.263 cm).

4. *Compute the moment of the vertical forces*

Thus, $M = W(h - a)$ = 407,700(0.055) = 22,400 ft·lb (30,374.4 N·m). Since $h > a$, the moment is righting. This constitutes the solution to part *a*. The remainder of this procedure is concerned with part *b*.

In Fig. 5, let M denote the point of intersection of the vertical line through B' and the line BG prolonged. Then M is termed the *metacenter* associated with this position, and the distance GM is called the *metacentric height.* Also BG is positive if G is above B, and GM is positive if M is above G. Thus, the moment of vertical forces is righting or upsetting depending on whether the metacentric height is positive or negative, respectively.

5. *Find the lever arm of the vertical forces*

Use the relation for metacentric height:

$$GM = \frac{I_{WL}}{V \cos \phi} - BG \qquad (7)$$

where I_{WL} = moment of inertia of original waterline section about axis through O. Or, $I_{WL} = (\frac{1}{12})(64)(20)^3$ = 42,670 ft^4 (368.3 m^4); GM = 42,670/6530 cos 6° − 6.05 = 0.52 ft (15.850 cm); $h - a$ = 0.52 sin 6° = 0.054 ft (1.646 cm), which agrees closely with the previous result.

Mechanics of Incompressible Fluids

The notational system is a = acceleration; A = area of stream cross section; C = discharge coefficient; D = diameter of pipe or depth of liquid in open channel; F = force; g = gravitational acceleration; H = total head, or total specific energy; h_F = loss of head between two sections caused by friction; h_L = total loss of head between two sections; h_V = difference in velocity heads at two sections if no losses occur; L = length of stream between two sections; M = mass of body; N_R = Reynolds number; p = pressure;

Q = volumetric rate of flow, or discharge; s = hydraulic gradient = $-dH/dL$; T = torque; V = velocity; w = specific weight; z = elevation above datum plane; ρ = density (mass per unit volume); μ = dynamic (or absolute) viscosity; ν = kinematic viscosity = μ/ρ; τ = shearing stress. The units used for each symbol are given in the calculation procedure where the symbol is used.

If the discharge of a flowing stream of liquid remains constant, the flow is termed *steady*. Let subscripts 1 and 2 refer to cross sections of the stream, 1 being the upstream section. From the definition of steady flow,

$$Q = A_1V_1 = A_2V_2 = \text{constant} \tag{8}$$

This is termed the *equation of continuity*. Where no statement is made to the contrary, it is understood that the flow is steady.

Conditions at two sections may be compared by applying the following equation, which is a mathematical statement of Bernoulli's theorem:

$$\frac{V_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{w} + z_2 + h_L \tag{9}$$

The terms on each side of this equation represent, in their order of appearance, the *velocity head, pressure head,* and *potential head* of the liquid. Alternatively, they may be considered to represent forms of specific energy, namely, kinetic, pressure, and potential energy.

The force causing a change in velocity is evaluated by applying the basic equation

$$F = Ma \tag{10}$$

Consider that liquid flows from section 1 to section 2 in a time interval t. At any instant, the volume of liquid bounded by these sections is Qt. The force required to change the velocity of this body of liquid from V_1 to V_2 is found from: $M = Qwt/g$; $a = (V_2 - V_1)/t$. Substituting in Eq. 10 gives $F = Qw(V_2 - V_1)/g$, or

$$F = \frac{A_1V_1w(V_2 - V_1)}{g} = \frac{A_2V_2w(V_2 - V_1)}{g} \tag{11}$$

VISCOSITY OF FLUID

Two horizontal circular plates 9 in. (228.6 mm) in diameter are separated by an oil film 0.08 in. (2.032 mm) thick. A torque of 0.25 ft·lb (0.339 N·m) applied to the upper plate causes that plate to rotate at a constant angular velocity of 4 revolutions per second (r/s) relative to the lower plate. Compute the dynamic viscosity of the oil.

Calculation Procedure:

1. *Develop equations for the force and torque*

Consider that the fluid film in Fig. 6*a* is in motion and that a fluid particle at boundary A has a velocity dV relative to a particle at B. The shearing stress in the fluid is

$$\tau = \mu \frac{dV}{dx} \tag{12}$$

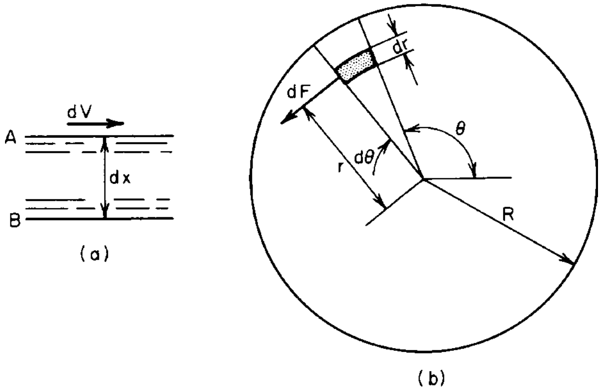


FIGURE 6

Figure 6*b* shows a cross section of the oil film, the shaded portion being an elemental surface. Let m = thickness of film; R = radius of plates; ω = angular velocity of one plate relative to the other; dA = area of elemental surface; dF = shearing force on elemental surface; dT = torque of dF with respect to the axis through the center of the plate.

Applying Eq. 12, develop these equations: $dF = 2\pi\omega\mu r^2\, dr\, d\theta/m$; $dT = r\, dF = 2\pi\omega\mu r^2\, dr\, d\theta/m$.

2. *Integrate the foregoing equation to obtain the resulting torque; solve for* μ

Thus,

$$\mu = \frac{Tm}{\pi^2 \omega R^4} \tag{13}$$

T = 0.25 ft·lb (0.339 N·m); m = 0.08 in. (2.032 mm); ω = 4 r/s; R = 4.5 in. (114.3 mm); $\mu = 0.25(0.08)(12)^3/[\pi^2(4)(4.5)^4]$ = 0.00214 lb·s/sq.ft. (0.1025 N·s/m^2).

APPLICATION OF BERNOULLI'S THEOREM

A steel pipe is discharging 10 ft^3/s (283.1 L/s) of water. At section 1, the pipe diameter is 12 in. (304.8 mm), the pressure is 18 lb/sq.in. (124.11 kPa), and the elevation is 140 ft (42.67 m). At section 2, farther downstream, the pipe diameter is 8 in. (203.2 mm), and the elevation is 106 ft (32.31 m). If there is a head loss of 9 ft (2.74 m) between these sections due to pipe friction, what is the pressure at section 2?

Calculation Procedure:

1. *Tabulate the given data*

Thus D_1 = 12 in. (304.8 mm); D_2 = 8 in. (203.2 mm); p_1 = 18 lb/sq.in. (124.11 kPa); p_2 = ?; z_1 = 140 ft (42.67 m); z_2 = 106 ft (32.31 m).

2. *Compute the velocity at each section*

Applying Eq. 8 gives $V_1 = 10/0.785 = 12.7$ ft/s (387.10 cm/s); $V_2 = 10/0.349 = 28.7$ ft/s (874.78 cm/s).

3. *Compute p_2 by applying Eq. 9*

Thus, $(p_2 - p_1)/w = (V_1^2 - V_2^2)/(2g) + z_1 - z_2 - h_F = (12.7^2 - 28.7^2)/64.4 + 140 - 106 - 9 = 14.7$ ft (448.06 cm); $p_2 = 14.7(62.4)/144 + 18 = 24.4$ lb/sq.in. (168.24 kPa).

FLOW THROUGH A VENTURI METER

A venturi meter of 3-in. (76.2-mm) throat diameter is inserted in a 6-in. (152.4-mm) diameter pipe conveying fuel oil having a specific gravity of 0.94. The pressure at the throat is 10 lb/sq.in. (68.95 kPa), and that at an upstream section 6 in. (152.4 mm) higher than the throat is 14.2 lb/sq.in. (97.91 kPa). If the discharge coefficient of the meter is 0.97, compute the flow rate in gallons per minute (liters per second).

Calculation Procedure:

1. *Record the given data, assigning the subscript 1 to the upstream section and 2 to the throat*

The loss of head between two sections can be taken into account by introducing a *discharge coefficient C.* This coefficient represents the ratio between the actual discharge Q and the discharge Q_i that would occur in the absence of any losses. Then $Q = CQ_i$, or $(V_2^2 - V_1^2)/(2g) = C^2 h_V$.

Record the given data: $D_1 = 6$ in. (152.4 mm); $p_1 = 14.2$ lb/sq.in. (97.91 kPa); $z_1 = 6$ in. (152.4 mm); $D_2 = 3$ in. (76.2 mm); $p_2 = 10$ lb/sq.in. (68.95 kPa); $z_2 = 0$; $C = 0.97$.

2. *Express V_1 in terms of V_2 and develop velocity and flow relations*

Thus,

$$V_2 = C\left[\frac{2gh_v}{(A_2/A_1)^2}\right]^{0.5} \tag{14a}$$

Also

$$Q = CA_2\left[\frac{2gh_v}{(A_2/A_1)^2}\right]^{0.5} \tag{14b}$$

If V_1 is negligible, these relations reduce to

$$V_2 = C(2gh_V)^{0.5} \tag{15a}$$

and

$$Q = CA_2(2gh_V)^{0.5} \tag{15b}$$

3. *Compute h_V by applying Eq. 9*

Thus, $h_V = (p_1 - p_2)/w + z_1 - z_2 = 4.2(144)/[0.94(62.4)] + 0.5 = 10.8$ ft (3.29 m).

4. *Compute Q by applying Eq. 14b*
Thus, $(A_2/A_1)^2 = (D_2/D_1)^4 = 1/16$; $A_2 = 0.0491$ sq.ft. (0.00456 m^2); and $Q = 0.97(0.049)[64.4 \times 10.8/(1 - 1/16)]^{0.5} = 1.30$ ft^3/s or, by using the conversion factor of 1 ft^3/s = 449 gal/min (28.32 L/s), the flow rate is 1.30(449) = 584 gal/min (36.84 L/s).

FLOW THROUGH AN ORIFICE

Compute the discharge through a 3-in. (76.2-mm) diameter square-edged orifice if the water on the upstream side stands 4 ft 8 in. (1.422 m) above the center of the orifice.

Calculation Procedure:

1. *Determine the discharge coefficient*
For simplicity, the flow through a square-edged orifice discharging to the atmosphere is generally computed by equating the area of the stream to the area of the opening and then setting the discharge coefficient $C = 0.60$ to allow for contraction of the issuing stream. (The area of the issuing stream is about 0.62 times that of the opening.)

2. *Compute the flow rate*
Since the velocity of approach is negligible, use Eq. 15*b*. Or, $Q = 0.60(0.0491)(64.4 \times 4.67)^{0.5} = 0.511$ ft^3/s (14.4675 L/s).

FLOW THROUGH THE SUCTION PIPE OF A DRAINAGE PUMP

Water is being evacuated from a sump through the suction pipe shown in Fig. 7. The entrance-end diameter of the pipe is 3 ft (91.44 cm); the exit-end diameter, 1.75 ft (53.34 cm). The exit pressure is 12.9 in. (32.77 cm) of mercury vacuum. The head loss at the entry is one-fifteenth of the velocity head at that point, and the head loss in the pipe due to friction is one-tenth of the velocity head at the exit. Compute the discharge flow rate.

Calculation Procedure:

1. *Convert the pressure head to feet of water*
The discharge may be found by comparing the conditions at an upstream point 1, where the velocity is negligible, with the conditions at point 3 (Fig. 7). Select the elevation of point 1 as the datum.

Converting the pressure head at point 3 to feet of water and using the specific gravity of mercury as 13.6, we have $p_3/w = -(12.9/12)13.6 = -14.6$ ft (−4.45 m).

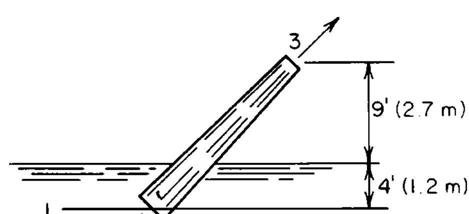


FIGURE 7

2. *Express the velocity head at 2 in terms of that at 3*
By the equation of continuity, $V_2 = A_3V_3/A_2 = (1.75/3)^2V_3 = 0.34V_3$.

3. *Evaluate V_3 by applying Eq. 9; then determine Q*

Thus, $V_1^2/(2g) + p_1/w = V_3^2/(2g) + p_3/w + z_3 + (^1/_{15})V_2^2/(2g) + (^1/_{10})V_3^2/(2g)$, or $0 + 4 + 0 = V_3^2/(2g) - 14.6 + 13 + [V_3^2/(2g)](^1/_{15} \times 0.34^2 = {}^1/_{10})$; $V_3 = 18.0$ ft/s (548.64 cm/s); then $Q_3 = A_3V_3 = 0.785(1.75)^2(18.0) = 43.3$ ft³/s (1225.92 L/s).

POWER OF A FLOWING LIQUID

A pump is discharging 8 ft³/s (226.5 L/s) of water. Gages attached immediately upstream and downstream of the pump indicate a pressure differential of 36 lb/sq.in. (248.2 kPa). If the pump efficiency is 85 percent, what is the horsepower output and input?

Calculation Procedure:

1. *Evaluate the increase in head of the liquid*

Power is the rate of performing work, or the amount of work performed in a unit time. If the fluid flows with a specific energy H, the total energy of the fluid discharged in a unit time is QwH. This expression thus represents the work that the flowing fluid can perform in a unit time and therefore the power associated with this discharge. Since 1 hp = 550 ft·lb/s,

$$1 \text{ hp} = \frac{QwH}{550} \tag{16}$$

In this situation, the power developed by the pump is desired. Therefore, H must be equated to the specific energy added by the pump.

To evaluate the increase in head, consider the differences of the two sections being considered. Since both sections have the same velocity and elevation, only their pressure heads differ. Thus, $p_2/w - p_1/w = 36(144)/62.4 = 83.1$ ft (2532.89 cm).

2. *Compute the horsepower output and input*

Thus, $\text{hp}_{\text{out}} = 8(62.4)(83.1)/550 = 75.4$ hp; $\text{hp}_{\text{in}} = 75.4/0.85 = 88.7$ hp.

DISCHARGE OVER A SHARP-EDGED WEIR

Compute the discharge over a sharp-edged rectangular weir 4 ft (121.9 cm) high and 10 ft (304.8 cm) long, with two end contractions, if the water in the canal behind the weir is 4 ft 9 in. (144.78 cm) high. Disregard the velocity of approach.

Calculation Procedure:

1. *Adopt a standard relation for this weir*

The discharge over a sharp-edged rectangular weir without end contractions in which the velocity of approach is negligible is given by the Francis formula as

$$Q = 3.33bh^{1.5} \tag{17a}$$

where b = length of crest and h = head on weir (i.e., the difference between the elevation of the crest and that of the water surface upstream of the weir).

2. *Modify the Francis equation for end contractions*

With two end contractions, the discharge of the weir is

$$Q = 3.33(b - 0.2h)h^{1.5} \tag{17b}$$

Substituting the given values yields $Q = 3.33(10 - 0.2 \times 0.75)0.75^{1.5} = 21.3$ ft³/s (603.05 L/s).

LAMINAR FLOW IN A PIPE

A tank containing crude oil discharges 340 gal/min (21.4 L/s) through a steel pipe 220 ft (67.1 m) long and 8 in. (203.2 mm) in diameter. The kinematic viscosity of the oil is 0.002 sq.ft./s (1.858 cm²/s). Compute the difference in elevation between the liquid surface in the tank and the pipe outlet.

Calculation Procedure:

1. *Identify the type of flow in the pipe*

To investigate the discharge in a pipe, it is necessary to distinguish between two types of fluid flow—*laminar* and *turbulent*. Laminar (or *viscous*) flow is characterized by the telescopic sliding of one circular layer of fluid past the adjacent layer, each fluid particle traversing a straight line. The velocity of the fluid flow varies parabolically from zero at the pipe wall to its maximum value at the pipe center, where it equals twice the mean velocity.

Turbulent flow is characterized by the formation of eddy currents, with each fluid particle traversing a sinuous path.

In any pipe the type of flow is ascertained by applying a dimensionless index termed the *Reynolds number*, defined as

$$N_R = \frac{DV}{\nu} \tag{18}$$

Flow is considered laminar if $N_R < 2100$ and turbulent if $N_R > 3000$.

In laminar flow the head loss due to friction is

$$h_F = \frac{32L\nu V}{gD^2} \tag{19a}$$

or

$$h_F = \left(\frac{64}{N_R}\right)\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \tag{19b}$$

Let 1 denote a point on the liquid surface and 2 a point at the pipe outlet. The elevation of 2 will be taken as datum.

To identify the type of flow, compute N_R. Thus, $D = 8$ in. (203.2 mm); $L = 220$ ft (6705.6 cm); $\nu = 0.002$ sq.ft./s (1.858 cm²/s); $Q = 340/449 = 0.757$, converting from gallons per minute to cubic feet per second. Then $V = Q/A = 0.757/0.349 = 2.17$ ft/s (66.142 cm/s). And $N_R = 0.667(2.17)/0.002 = 724$. Therefore, the flow is laminar because N_R is less than 2100.

2. *Express all losses in terms of the velocity head*

By Eq. 19*b*, $h_F = (64/724)(220/0.667)V^2/(2g) = 29.2V^2/(2g)$. Where $L/D > 500$, the following may be regarded as negligible in comparison with the loss due to friction: loss at pipe entrance, losses at elbows, velocity head at the discharge, etc. In this instance, include the secondary items. The loss at the pipe entrance is $h_E = 0.5V^2/(2g)$. The total loss is $h_L = 29.7V^2/(2g)$.

3. *Find the elevation of 1 by applying Eq. 9*

Thus, $z_1 = V_2^2/(2g) + h_L = 30.7V_2^2/(2g) = 30.7(2.17)^2/64.4 = 2.24$ ft (68.275 cm).

TURBULENT FLOW IN PIPE—APPLICATION OF DARCY-WEISBACH FORMULA

Water is pumped at the rate of 3 ft³/s (85.0 L/s) through an 8-in. (203.2-mm) fairly smooth pipe 2600 ft (792.48 m) long to a reservoir where the water surface is 180 ft (50.86 m) higher than the pump. Determine the gage pressure at the pump discharge.

Calculation Procedure:

1. *Compute h_F*

Turbulent flow in a pipe flowing full may be investigated by applying the Darcy-Weisbach formula for friction head

$$h_F = \frac{fLV^2}{2gD} \tag{20}$$

where f is a friction factor. However, since the friction head does not vary precisely in the manner implied by this equation, f is dependent on D and V, as well as the degree of roughness of the pipe. Values of f associated with a given set of values of the independent quantities may be obtained from Fig. 8.

Accurate equations for h_F are the following:

Extremely smooth pipes:

$$h_F = \frac{0.30LV^{1.75}}{1000D^{1.25}} \tag{21a}$$

Fairly smooth pipes:

$$h_F = \frac{0.38LV^{1.86}}{1000D^{1.25}} \tag{21b}$$

Rough pipes:

$$h_F = \frac{0.50LV^{1.95}}{1000D^{1.25}} \tag{21c}$$

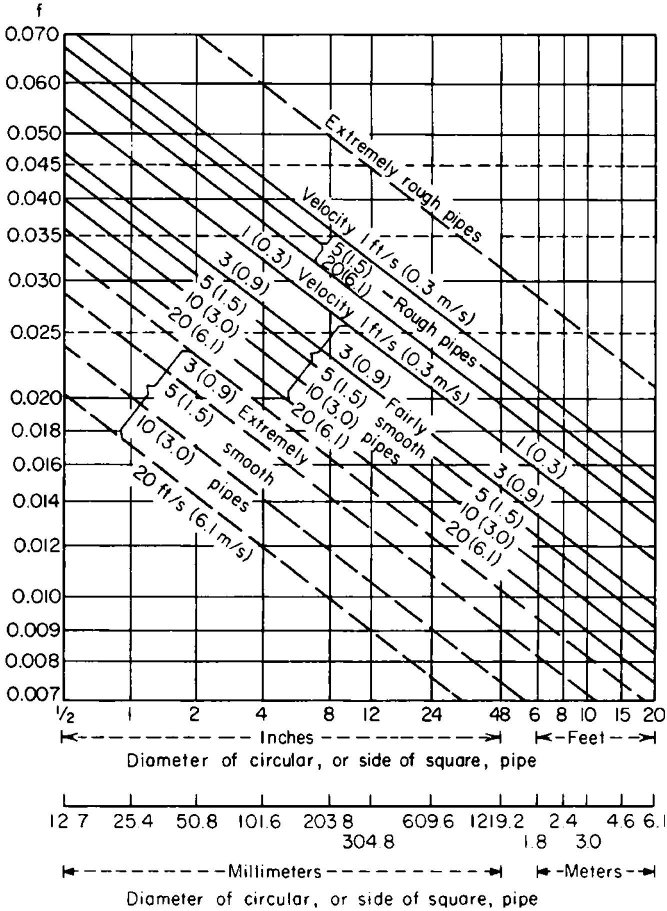


FIGURE 8. Flow of water in pipes. (*From E. W. Schoder and F. M. Dawson, Hydraulics, McGraw-Hill Book Company, New York, 1934. By permission of the publishers.*)

Extremely rough pipes:

$$h_F = \frac{0.69LV^2}{1000D^{1.25}} \tag{21d}$$

Using Eq. 21*b* gives $V = Q/A = 3/0.349 = 8.60$ ft/s (262.128 cm/s); $h_F = 0.38(2.6)(8.60)^{1.86}/0.667^{1.25} = 89.7$ ft (27.34 m).

2. *Alternatively, determine h_F using Eq. 20*

First obtain the appropriate f value from Fig. 8, or $f = 0.020$ for this pipe. Then $h_F = 0.020(2.600/0.667)(8.60^2/64.4) = 89.6$ ft (27.31 m).

3. *Compute the pressure at the pump discharge*

Use Eq. 9. Since $L/D > 500$, ignore the secondary items. Then $p_1/w = z_2 + h_F = 180 + 89.6 = 269.6$ ft (82.17 m), $p_1 = 269.6(62.4)/144 = 117$ lb/sq.in. (806.7 kPa).

DETERMINATION OF FLOW IN A PIPE

Two reservoirs are connected by a 7000-ft (2133.6-m) fairly smooth cast-iron pipe 10 in. (254.0 mm) in diameter. The difference in elevation of the water surfaces is 90 ft (27.4 m). Compute the discharge to the lower reservoir.

Calculation Procedure:

1. *Determine the fluid velocity and flow rate*

Since the secondary items are negligible, the entire head loss of 90 ft (27.4 m) results from friction. Using Eq. 21*b* and solving for V, we have $90 = 0.38(7)V^{1.86}/0.833^{1.25}$; $V = 5.87$ ft/s (178.918 cm/s). Then $Q = VA = 5.87(0.545) = 3.20$ ft^3/s (90.599 L/s).

2. *Alternatively, assume a value of f and compute V*

Referring to Fig. 8, select a value for f. Then compute V by applying Eq. 20. Next, compare the value of f corresponding to this result with the assumed value of f. If the two values differ appreciably, assume a new value of f and repeat the computation. Continue this process until the assumed and actual values of f agree closely.

PIPE-SIZE SELECTION BY THE MANNING FORMULA

A cast-iron pipe is to convey water at 3.3 ft^3/s (93.430 L/s) on a grade of 0.001. Applying the Manning formula with $n = 0.013$, determine the required size of pipe.

Calculation Procedure:

1. *Compute the pipe diameter*

The Manning formula, which is suitable for both open and closed conduits, is

$$V = \frac{1.486R^{2/3}s^{1/2}}{n} \tag{22}$$

where n = roughness coefficient; R = hydraulic radius = ratio of cross-sectional area of pipe to the wetted perimeter of the pipe; s = hydraulic gradient = dH/dL. If the flow is uniform, i.e., the area and therefore the velocity are constant along the stream, then the loss of head equals the drop in elevation, and the grade of the conduit is s.

For a circular pipe flowing full, Eq. 22 becomes

$$D = \left(\frac{2.159Qn}{s^{1/2}}\right)^{3/8} \tag{22a}$$

Substituting numerical values gives $D = (2.159 \times 3.3 \times 0.013/0.001^{1/2})^{3/8} = 1.50$ ft (45.72 cm). Therefore, use an 18-in. (457.2-mm) diameter pipe.

LOSS OF HEAD CAUSED BY SUDDEN ENLARGEMENT OF PIPE

Water flows through a pipe at 4 ft^3/s (113.249 L/s). Compute the loss of head resulting from a change in pipe size if (*a*) the pipe diameter increases abruptly from 6 to 10 in. (152.4 to 254.0 mm); (*b*) the pipe diameter increases abruptly from 6 to 8 in. (152.4 to 203.2 mm) at one section and then from 8 to 10 in. (203.2 to 254.0 mm) at a section farther downstream.

Calculation Procedure:

1. *Evaluate the pressure-head differential required to decelerate the liquid*

Where there is an abrupt increase in pipe size, the liquid must be decelerated upon entering the larger pipe, since the fluid velocity varies inversely with area. Let subscript 1 refer to a section immediately downstream of the enlargement, where the higher velocity prevails, and let subscript 2 refer to a section farther downstream, where deceleration has been completed. Disregard the frictional loss.

Using Eq. 11 we see $p_2/w = p_1/w + (V_1V_2 - V_2^2)/g$.

2. *Combine the result of step 1 with Eq. 9*

The result is Borda's formula for the head loss h_E caused by sudden enlargement of the pipe cross section:

$$h_E = \frac{(V_1 - V_2)^2}{2g} \tag{23}$$

As this investigation shows, only part of the drop in velocity head is accounted for by a gain in pressure head. The remaining head h_E is dissipated through the formation of eddy currents at the entrance to the larger pipe.

3. *Compute the velocity in each pipe*

Thus

Pipe diam, in. (mm)	Pipe area, sq.ft. (m^2)	Fluid velocity, ft/s (cm/s)
6 (152.4)	0.196 (0.0182)	20.4 (621.79)
8 (203.2)	0.349 (0.0324)	11.5 (350.52)
10 (254.0)	0.545 (0.0506)	7.3 (222.50)

4. *Find the head loss for part a*

Thus, $h_E = (20.4 - 7.3)^2/64.4 = 2.66$ ft (81.077 cm).

5. *Find the head loss for part b*

Thus, $h_E = [(20.4 - 11.5)^2 + (11.5 - 7.3)^2]/64.4 = 1.50$ ft (45.72 cm). Comparison of these results indicates that the eddy-current loss is attenuated if the increase in pipe size occurs in steps.

DISCHARGE OF LOOPING PIPES

A pipe carrying 12.5 ft³/s (353.90 L/s) of water branches into three pipes of the following diameters and lengths; $D_1 = 6$ in. (152.4 mm); $L_1 = 1000$ ft (304.8 m); $D_2 = 8$ in. (203.2 mm); $L_2 = 1300$ ft (396.2 m); $D_3 = 10$ in. (254.0 mm); $L_3 = 1200$ ft (365.8 m). These pipes rejoin at their downstream ends. Compute the discharge in the three pipes, considering each as fairly smooth.

Calculation Procedure:

1. *Express Q as a function of D and L*

Since all fluid particles have the same energy at the juncture point, irrespective of the loops they traversed, the head losses in the three loops are equal. The flow thus divides itself in a manner that produces equal values of h_F in the loops.

Transforming Eq. 21*b*,

$$Q = \frac{kD^{2.67}}{L^{0.538}} \tag{24}$$

where k is a constant.

2. *Establish the relative values of the discharges; then determine the actual values*

Thus, $Q_2/Q_1 = (8/6)^{2.67}/1.3^{0.538} = 1.87$; $Q_3/Q_1 = (10/6)^{2.67}/1.2^{0.538} = 3.55$. Then $Q_1 + Q_2 + Q_3 = Q_1(1 + 1.87 + 3.55) = 12.5$ ft³/s (353.90 L/s). Solving gives $Q_1 = 1.95$ ft³/s (55.209 L/s); $Q_2 = 3.64$ ft³/s (103.056 L/s); $Q_3 = 6.91$ ft³/s (195.637 L/s).

FLUID FLOW IN BRANCHING PIPES

The pipes *AM*, *MB*, and *MC* in Fig. 9 have the diameters and lengths indicated. Compute the water flow in each pipe if the pipes are considered rough.

Calculation Procedure:

1. *Write the basic equations governing the discharges*

Let subscripts 1, 2, and 3 refer to *AM*, *MB*, and *MC*, respectively. Then $h_{F1} + h_{F2} = 110$; $h_{F1} + h_{F3} = 150$, Eq. *a*; $Q_1 = Q_2 + Q_3$, Eq. *b*.

2. *Transform Eq. 21c*

The transformed equation is

$$Q = 38.7D^{2.64}\left(\frac{h_F}{L}\right)^{0.513} \tag{25}$$

3. *Assume a trial value for h_{F1} and find the discharge; test the result*

Use Eqs. *a* and 25 to find the discharges. Test the results for compliance with Eq. *b*. If we assume $h_{F1} = 70$ ft (21.3 m), then $h_{F2} = 40$ ft (12.2 m) and $h_{F3} = 80$ ft (24.4 m); $Q_1 =$

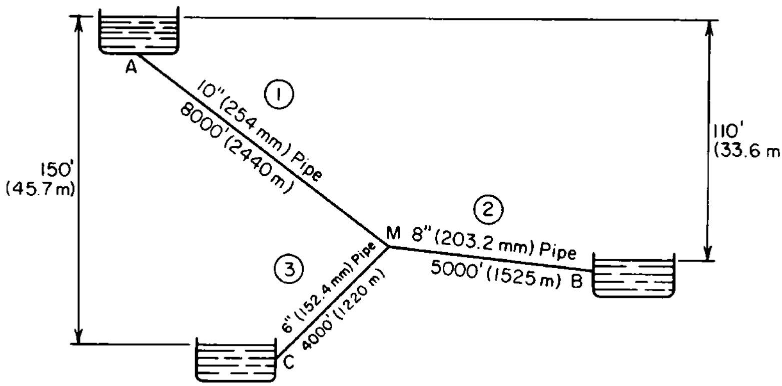


FIGURE 9. Branching pipes.

$38.7(0.833)^{2.64}(70/8000)^{0.513} = 2.10$ ft^3/s (59.455 L/s). Similarly, $Q_2 = 1.12$ ft^3/s (31.710 L/s) and $q_3 = 0.83$ ft^3/s (23.499 L/s); $Q_2 + Q_3 = 1.95 < Q_1$. The assumed value of h_{F1} is excessive.

4. ***Make another assumption for h_{F1} and the corresponding revisions***
Assume $h_{F1} = 66$ ft (20.1 m). Then $Q_1 = 2.10(66/70)^{0.513} = 2.04$ ft^3/s (57.757 L/s). Similarly, $Q_2 = 1.18$ ft^3/s (33.408 L/s); $Q_3 = 0.85$ ft^3/s (24.065 L/s). $Q_2 + Q_3 = 2.03$ ft^3/s (57.736 L/s). These results may be accepted as sufficiently precise.

UNIFORM FLOW IN OPEN CHANNEL—DETERMINATION OF SLOPE

It is necessary to convey 1200 ft^3/s (33,974.6 L/s) of water from a dam to a power plant in a canal of rectangular cross section, 24 ft (7.3 m) wide and 10 ft (3.0 m) deep, having a roughness coefficient of 0.016. The canal is to flow full. Compute the required slope of the canal in feet per mile (meters per kilometer).

Calculation Procedure:

1. ***Apply Eq. 22***
Thus, $A = 24(10) = 240$ sq.ft. (22.3 m^2); wetted perimeter = WP = 24 + 2(10) = 44 ft (13.4 m); $R = 240/44 = 5.45$ ft (1.661 m); $V = 1200/240 = 5$ ft/s (152.4 cm/s); $s = [nV/(1.486R^{2/3})]^2 = [0.016 \times 5/(1.486 \times 5.45^{2/3})]^2 = 0.000302$; slope = 0.000302(5280 ft/mi) = 1.59 ft/mi (0.302 m/km).

REQUIRED DEPTH OF CANAL FOR SPECIFIED FLUID FLOW RATE

A trapezoidal canal is to carry water at 800 ft^3/s (22,649.7 L/s). The grade of the canal is 0.0004; the bottom width is 25 ft (7.6 m); the slope of the sides is $1\frac{1}{2}$ horizontal to

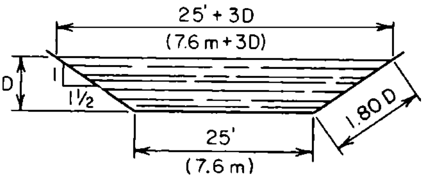


FIGURE 10

1 vertical; the roughness coefficient is 0.014. Compute the required depth of the canal, to the nearest tenth of a foot.

Calculation Procedure:

1. ***Transform Eq. 22 and compute $AR^{2/3}$***

Thus, $AR^{2/3} = nQ/(1.486s^{1/2})$, Eq. 22*b*. Or, $AR^{2/3} = 0.014(800)/[1.486(0.0004)^{1/2}] = 377$.

2. ***Express the area and wetted perimeter in terms of D (Fig. 10)***

Side of canal $= D(1^2 + 1.5^2)^{0.5} = 1.80D$. $A = D(25 + 1.5D)$; WP $= 25 + 360D$.

3. ***Assume the trial values of D until Eq. 22b is satisfied***

Thus, assume $D = 5$ ft (152.4 cm); $A = 162.5$ sq.ft. (15.10 m^2); WP $= 43$ ft (1310.6 cm); $R = 3.78$ ft (115.2 cm); $AR^{2/3} = 394$. The assumed value of D is therefore excessive because the computed $AR^{2/3}$ is greater than the value computed in step 1.

Next, assume a lower value for D, or $D = 4.9$ ft (149.35 cm); $A = 158.5$ sq.ft. (14.72 m^2); WP $= 42.64$ ft (1299.7 cm); $R = 3.72$ ft (113.386 cm); $AR^{2/3} = 381$. This is acceptable. Therefore, $D = 4.9$ ft (149.35 cm).

ALTERNATE STAGES OF FLOW; CRITICAL DEPTH

A rectangular channel 20 ft (609.6 cm) wide is to discharge 500 ft^3/s (14,156.1 L/s) of water having a specific energy of 4.5 ft·lb/lb (1.37 J/N). (*a*) Using $n = 0.013$, compute the required slope of the channel. (*b*) Compute the maximum potential discharge associated with the specific energy of 4.5 ft·lb/lb (1.37 J/N). (*c*) Compute the minimum of specific energy required to maintain a flow of 500 ft^3/s (14,156.1 L/s).

Calculation Procedure:

1. ***Evaluate the specific energy of an elemental mass of liquid at a distance z above the channel bottom***

To analyze the discharge conditions at a given section in a channel, it is advantageous to evaluate the specific energy (or head) by taking the elevation of the bottom of the channel *at the given section* as datum. Assume a uniform velocity across the section, and let $D =$ depth of flow, ft (cm); $H_e =$ specific energy as computed in the prescribed manner; $Q_u =$ discharge through a unit width of channel, $ft^3(s{\cdot}ft)$ [L/(s·cm)].

Evaluating the specific energy of an elemental mass of liquid at a given distance z above the channel bottom, we get

$$H_e = \frac{Q_u^2}{2gD^2} + D \tag{26}$$

Thus, H_e is constant across the entire section. Moreover, if the flow is uniform, as it is here, H_e is constant along the entire stream.

2. *Apply the given values and solve for D*

Thus, H_e = 4.5 ft·lb/lb (1.37 J/N); Q_u = 500/20 = 25 ft^3/(s·ft) [2323 L/(s·m)]. Rearrange Eq. 26 to obtain

$$D^2(H_e - D) = \frac{Q_u^2}{2g} \qquad (26a)$$

Or, $D^2(4.5 - D) = 25^2/64.4 = 9.705$. This cubic equation has two positive roots, D = 1.95 ft (59.436 cm) and D = 3.84 ft (117.043 cm). There are therefore two stages of flow that accommodate the required discharge with the given energy. [The third root of the equation is $D = -1.29$ ft (−39.319 cm), an impossible condition.]

3. *Compute the slope associated with the computed depths*

Using Eq. 22, at the lower stage we have D = 1.95 ft (59.436 cm); A = 20(1.95) = 39.0 sq.ft. (36,231.0 cm^2); WP = 20 + 2(1.95) = 23.9 ft (728.47 cm); R = 39.0/23.9 = 1.63 ft (49.682 cm); V = 25/1.95 = 12.8 ft/s (390.14 cm/s); $s = [nV/(1.486R^{2/3})]^2 = (0.013 \times 12.8/1.486 \times 1.63^{2/3})^2 = 0.00654$.

At the upper stage D = 3.84 ft (117.043 cm); A = 20(3.84) = 76.8 sq.ft. (71,347.2 cm^2); WP = 20 + 2(3.84) = 27.68 ft (843.686 cm); R = 76.8/27.68 = 2.77 ft (84.430 cm); V = 25/3.84 = 6.51 ft/s (198.4 cm/s); $s = [0.013 \times 6.51/(1.486 \times 2.77^{2/3})]^2 = 0.000834$. This constitutes the solution to part *a*.

4. *Plot the D-Q_u curve*

For part *b*, consider H_e as remaining constant at 4.5 ft·lb/lb (1.37 J/N) while Q_u varies. Plot the D-Q_u curve as shown in Fig. 11*a*. The depth that provides the maximum potential discharge is called the *critical depth* with respect to the given specific energy.

5. *Differentiate Eq. 26 to find the critical depth; then evaluate $Q_{u,max}$*

Differentiating Eq. 26 and setting $dQ_u/dD = 0$ yield

$$\text{Critical depth } D_c = {}^2\!/\!_3 H_e \qquad (27)$$

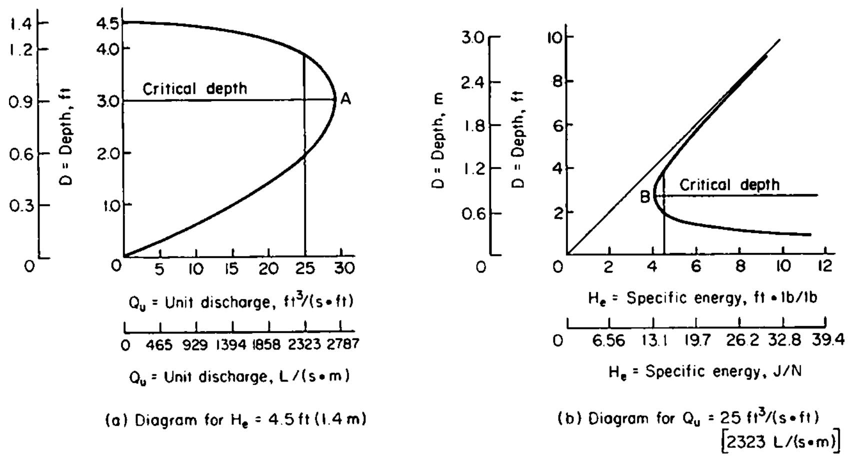


FIGURE 11

Or, $D_c = \frac{2}{3}(4.5) = 3.0$ ft (91.44 cm); $Q_{u,\max} = [64.4(4.5 \times 3.0^2 - 3.0^2)]^{0.5} = 29.5$ ft³/(s·ft) [2741 L/(s·m)]; $Q_{\max} = 29.5(20) = 590$ ft³/s (16,704.2 L/s). This constitutes the solution to part *b*.

6. *Plot the D-H_e curve*

For part *c*, consider Q_u as remaining constant at 25 ft³/(s·ft) [2323 L/(s·m)] while H_c varies. Plot the D-H_e curve as shown in Fig. 11*b*. (This curve is asymptotic with the straight lines $D = H_e$ and $D = 0$.) The depth at which the specific energy is minimum is called the *critical depth* with respect to the given unit discharge.

7. *Differentiate Eq. 26 to find the critical depth; then evaluate $H_{e,min}$*

Differentiating gives

$$D_c = \left(\frac{Q_u^2}{g}\right)^{1/3} \tag{28}$$

Then $D_c = (25^2/32.2)^{1/3} = 2.69$ ft (81.991 cm). Then $H_{e,\min} = 25^2/[64.4(2.69)^2] + 2.69 = 4.03$ ft·lb/lb (1.229 J/N).

The values of D as computed in part *a* coincide with those obtained by referring to the two graphs in Fig. 11. The equations derived in this procedure are valid solely for rectangular channels, but analogous equations pertaining to other channel profiles may be derived in a similar manner.

DETERMINATION OF HYDRAULIC JUMP

Water flows over a 100-ft (30.5-m) long dam at 7500 ft³/s (212,400 L/s). The depth of tailwater on the level apron is 9 ft (2.7 m). Determine the depth of flow immediately upstream of the hydraulic jump.

Calculation Procedure:

1. *Find the difference in hydrostatic forces per unit width of channel required to decelerate the liquid*

Refer to Fig. 12. *Hydraulic jump* designates an abrupt transition from lower-stage to upper-stage flow caused by a sharp decrease in slope, sudden increase in roughness,

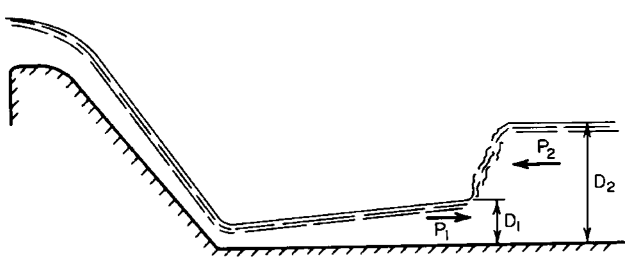


FIGURE 12. Hydraulic jump on apron of dam.

encroachment of backwater, or some other factor. The deceleration of liquid requires an increase in hydrostatic pressure, but only part of the drop in velocity head is accounted for by a gain in pressure head. The excess head is dissipated in the formation of a turbulent standing wave. Thus, the phenomenon of hydraulic jump resembles the behavior of liquid in a pipe at a sudden enlargement, as analyzed in an earlier calculation procedure.

Let D_1 and D_2 denote the depth of flow immediately upstream and downstream of the jump, respectively. Then $D_1 < D_c < D_2$. Refer to Fig. 11*b*. Since the hydraulic jump is accompanied by a considerable drop in energy, the point on the D-H_e diagram that represents D_2 lies both above and to the left of that representing D_1. Therefore, the upstream depth is less than the depth that would exist in the absence of any loss.

Using literal values, apply Eq. 11 to find the difference in hydrostatic forces per unit width of channel that is required to decelerate the liquid. Solve the resulting equation for D_1:

$$D_1 = -\frac{D_2}{2} + \left(\frac{2V_2^2 D_2}{g} + \frac{D_2^2}{4}\right)^{0.5} \tag{29}$$

2. *Substitute numerical values in Eq. 29*

Thus, $Q_u = 7500/100 = 75$ ft³/(s·ft) [6969 L/(s·m)]; $V_2 = 75/9 = 8.33$ ft/s (2.538 m/s); $D_1 = -9/2 + (2 \times 8.33^2 \times 9/32.2 + 9^2/4)^{0.5} = 3.18$ ft (0.969 m).

RATE OF CHANGE OF DEPTH IN NONUNIFORM FLOW

The unit discharge in a rectangular channel is 28 ft³/(s·ft) [2602 L/(s·m)]. The energy gradient is 0.0004, and the grade of the channel bed is 0.0010. Determine the rate at which the depth of flow is changing in the downstream direction (i.e., the grade of the liquid surface with respect to the channel bed) at a section where the depth is 3.2 ft (0.97 m).

Calculation Procedure:

1. *Express H as a function of D*

Let H = total specific energy at a given section as evaluated by selecting a fixed horizontal reference plane; L = distance measured in downstream direction; z = elevation of given section with respect to datum plane; s_b = grade of channel bed $= -dz/dL$; s_e = energy gradient $= -dH/dL$.

Express H as a function of D by annexing the potential-energy term to Eq. 26. Thus,

$$H = \frac{Q_u^2}{2gD^2} + D + z \tag{30}$$

2. *Differentiate this equation with respect to L to obtain the rate of change of D; substitute numerical values*

Differentiating gives

$$\frac{dD}{dL} = \frac{s_b - s_e}{1 - Q_u^2/(gD^3)} \qquad (31a)$$

or in accordance with Eq. 28,

$$\frac{dD}{dL} = \frac{s_b - s_e}{1 - d_c^3/D^3} \qquad (31b)$$

Substituting yields $Q_u^2/(gD^3) = 28^2/(32.2 \times 3.2^3) = 0.743$; $dD/dL = (0.0010 - 0.0004)/(1 - 0.743) = 0.00233$ ft/ft (0.00233 m/m). The depth is increasing in the downstream direction, and the water is therefore being decelerated.

As Eq. 31*b* reveals, the relationship between the actual depth at a given section and the critical depth serves as a criterion in ascertaining whether the depth is increasing or decreasing.

DISCHARGE BETWEEN COMMUNICATING VESSELS

In Fig. 13, liquid is flowing from tank *A* to tank *B* through an orifice near the bottom. The area of the liquid surface is 200 sq.ft. (18.58 m^2) in *A* and 150 sq.ft. (13.93 m^2) in *B*. Initially, the difference in water levels is 14 ft (4.3 m), and the discharge is 2 ft^3/s (56.6 L/s). Assuming that the discharge coefficient remains constant, compute the time required for the water level in tank *A* to drop 1.8 ft (0.54 m).

Calculation Procedure:

1. *By expressing the change in h during an elemental time interval, develop the time-interval equation*

Let A_a and A_b denote the area of the liquid surface in tanks *A* and *B*, respectively; let subscripts 1 and 2 refer to the beginning and end, respectively, of a time interval *t*. Then

$$t = \frac{2A_a A_b (h_1 - [h_1 h_2]^{0.5})}{Q_1 (A_a + A_b)} \qquad (32)$$

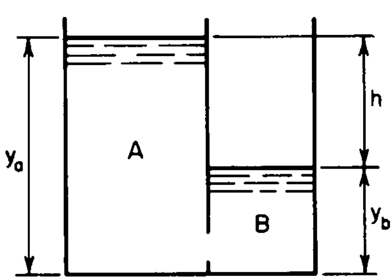


FIGURE 13

2. *Find the value of h when y_a diminishes by 1.8 ft (0.54 m)*

Thus, $\Delta y_b = (-A_a/A_b)(\Delta y_a) = -(200/150)(-1.8) = 2.4$ ft (0.73 m); $\Delta h = \Delta y_a - \Delta y_b = -1.8 - 2.4 = -4.2$ ft (−1.28 m); $h_1 = 14$ ft (4.3 m); $h_2 = 14 - 4.2 = 9.8$ ft (2.99 m).

3. *Substitute numerical values in Eq. 32 and solve for t*

Thus, $t = 2(200)(150)[14 - (14 \times 9.8)^{0.5}]/[2(200 + 150)] = 196$ s $= 3.27$ min.

VARIATION IN HEAD ON A WEIR WITHOUT INFLOW TO THE RESERVOIR

Water flows over a weir of 60-ft (18.3-m) length from a reservoir having a surface area of 50 acres (202,350 m^2). If the inflow to the reservoir ceases when the head on the weir is 2 ft (0.6 m), what will the head be at the expiration of 1 h? Consider that the instantaneous discharge is given by Eq. 17*a*.

Calculation Procedure:

1. *Develop the time-interval equation*

Let A = surface area of reservoir and C = numerical constant in discharge equation; and subscripts 1 and 2 refer to the beginning and end, respectively, of a time interval t. By expressing the change in head during an elemental time interval,

$$t = \frac{2A}{Cb(1/h_2^{0.5} - 1/h_1^{0.5})} \tag{33}$$

2. *Substitute numerical values in Eq. 33; solve for h_2*

Thus, $A = 50(43{,}560) = 2{,}178{,}000$ sq.ft. (202,336.2 m^2); $t = 3600$ s; solving gives $h_2 =$ 1.32 ft (0.402 m).

VARIATION IN HEAD ON A WEIR WITH INFLOW TO THE RESERVOIR

Water flows over an 80-ft (24.4-m) long weir from a reservoir having a surface area of 6,000,000 sq.ft. (557,400.0 m^2) while the rate of inflow to the reservoir remains constant at 2175 ft^3/s (61,578.9 L/s). How long will it take for the head on the weir to increase from zero to 95 percent of its maximum value? Consider that the instantaneous rate of flow over the weir is $3.4bh^{1.5}$.

Calculation Procedure:

1. *Compute the maximum head on the weir by equating outflow to inflow*

The water in the reservoir reaches its maximum height when equilibrium is achieved, i.e., when the rate of outflow equals the rate of inflow. Let Q_i = rate of inflow; Q_o = rate of outflow at a given instant; t = time elapsed since the start of the outflow.

Equating outflow to inflow yields $3.4(80h_{\max}^{1.5}) = 2175$; $h_{\max} = 4.0$ ft (1.2 m); $0.95h_{\max} = 3.8$ ft (1.16 m).

2. *Using literal values, determine the time interval dt during which the water level rises a distance dh*

Thus, with C = numerical constant in the discharge equation,

$$dt = \frac{A}{Q_i - Cbh^{1.5}}\, dh \tag{34}$$

The right side of this equation is not amenable to direct integration. Consequently, the only feasible way of computing the time is to perform an approximate integration.

3. *Obtain the approximate value of the required time*

Select suitable increments of h, calculate the corresponding increments of t, and total the latter to obtain an approximate value of the required time. In calculating Q_o, apply the mean value of h associated with each increment.

The precision inherent in the result thus obtained depends on the judgment used in selecting the increments of h, and a clear visualization of the relationship between h and t is essential. Let $m = dt/dh = A/(Q_i - Cbh^{1.5})$. The m-h curve is shown in Fig. 14a. Then, $t = \int m\, dh$ = area between the m-h curve and h axis.

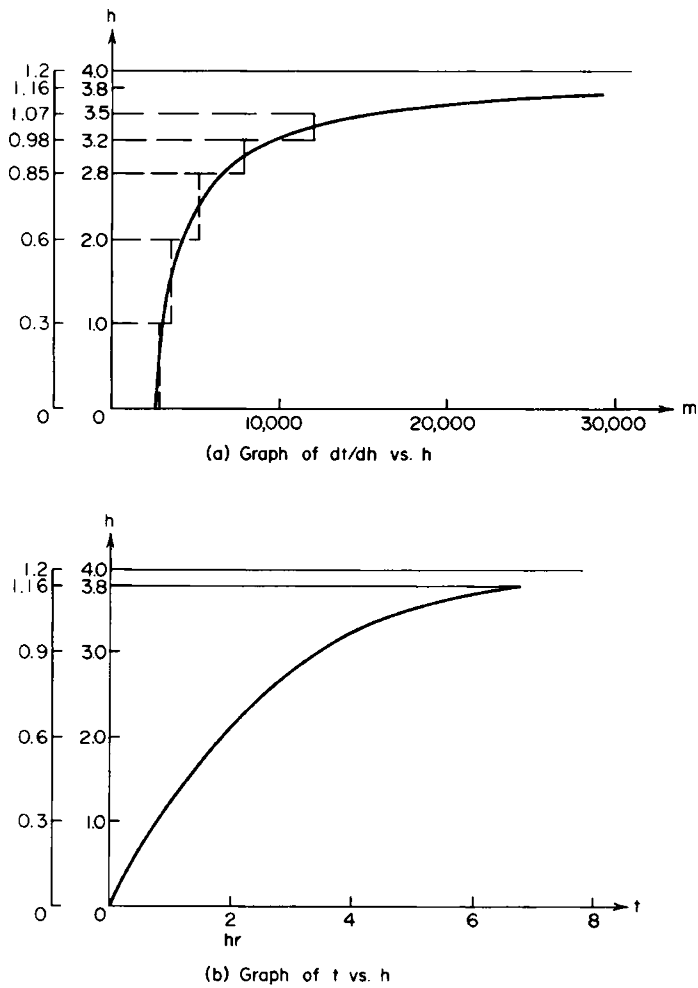


FIGURE 14

TABLE 1. Approximate Integration of Eq. 34

Δh, ft (m)	h_b, ft (m)	h_m, ft (m)	m, s/ft (s/m)	Δt, s
1.0 (0.30)	0 (0.00)	0.5 (0.15)	2,890 (9,633.3)	2,890
1.0 (0.30)	1.0 (0.30)	1.5 (0.46)	3,580 (11,933.3)	3,580
0.8 (0.24)	2.0 (0.61)	2.4 (0.73)	5,160 (17,308.3)	4,130
0.4 (0.12)	2.8 (0.85)	3.0 (0.91)	7,870 (26,250.0)	3,150
0.3 (0.09)	3.2 (0.98)	3.35 (1.02)	11,830 (39,444.4)	3,550
0.2 (0.06)	3.5 (1.07)	3.6 (1.10)	18,930 (63,166.7)	3,790
0.1 (0.03)	3.7 (1.13)	3.75 (1.14)	30,000 (100,000)	3,000
Total				24,090

This area is approximated by summing the areas of the rectangles as indicated in Fig. 14, the length of each rectangle being equal to the value of m at the center of the interval. Note that as h increases, the increments Δh should be made progressively smaller to minimize the error introduced in the procedure.

Select the increments shown in Table 1, and perform the indicated calculations. The symbols h_b and h_m denote the values of h at the beginning and center, respectively, of an interval. The following calculations for the third interval illustrate the method: $h_m = \frac{1}{2}(2.0 + 2.8) = 2.4$ ft (0.73 m); $m = 6{,}000{,}000/(2175 - 3.4 \times 80 \times 2.4^{1.5}) = 5160$ s/ft (16,929.1 s/m); $\Delta t = m\,\Delta h = 5160(0.8) = 4130$ s. From Table 1 the required time is $t = 24{,}090$ s $=$ 6 h 41.5 min.

The *t-h* curve is shown in Fig. 14*b*. The time required for the water to reach its maximum height is difficult to evaluate with precision because m becomes infinitely large as h approaches h_{max}; that is, the water level rises at an imperceptible rate as it nears its limiting position.

DIMENSIONAL ANALYSIS METHODS

The velocity of a raindrop in still air is known or assumed to be a function of these quantities: gravitational acceleration, drop diameter, dynamic viscosity of the air, and the density of both the water and the air. Develop the dimensionless parameters associated with this phenomenon.

Calculation Procedure:

1. *Using a generalized notational system, record the units in which the six quantities of this situation are expressed*

Dimensional analysis is an important tool both in theoretical investigations and in experimental work because it clarifies the relationships intrinsic in a given situation.

A quantity that appears in every dimensionless parameter is termed *repeating*; a quantity that appears in only one parameter is termed *nonrepeating.* Since the engineer is usually more accustomed to dealing with units of force rather than of mass, the force-length-time system of units is applied here. Let F, L, and T denote units of force, length, and time, respectively.

By using this generalized notational system, it is convenient to write the appropriate USCS units and then replace these with the general units. For example, with respect to acceleration: USCS units, ft/s^2; general units, L/T^2 or LT^{-2}. Similarly, with respect to density (w/g): USCS units, (lb/ft^3)/(ft/s^2); general units, FL^{-3}/LT^{-2} or $FL^{-4}T^2$.

The results are shown in the following table.

Quantity	Units
V = velocity of raindrop	LT^{-1}
g = gravitational acceleration	LT^{-2}
D = diameter of drop	L
μ_a = air viscosity	$FL^{-2}T$
ρ_w = water density	$FL^{-4}T^2$
ρ_a = air density	$FL^{-4}T^2$

2. *Compute the number of dimensionless parameters present*

This phenomenon contains six physical quantities and three units. Therefore, as a consequence of Buckingham's pi theorem, the number of dimensionless parameters is $6 - 3 = 3$.

3. *Select the repeating quantities*

The number of repeating quantities must equal the number of units (three here). These quantities should be independent, and they should collectively contain all the units present. The quantities g, D, and μ_a satisfy both requirements and therefore are selected as the repeating quantities.

4. *Select the dependent variable V as the first nonrepeating quantity*

Then write $\pi_1 = g^x D^y \mu_a^z V$, Eq. *a*, where π_1 is a dimensionless parameter and x, y, and z are unknown exponents that may be evaluated by experiment.

5. *Transform Eq. a to a dimensional equation*

Do this by replacing each quantity with the units in which it is expressed. Then perform the necessary expansions and multiplications. Or, $F^0L^0T^0 = (LT^{-2})^x L^y \times (FL^{-2}T)^z LT^{-1}$, $F^0L^0T^0 = F^z L^{x+y-2z+1} T^{-2x+z-1}$, Eq. *b*.

Every equation must be dimensionally homogeneous; i.e., the units on one side of the equation must be consistent with those on the other side. Therefore, the exponent of a unit on one side of Eq. *b* must equal the exponent of that unit on the other side.

6. *Evaluate the exponents x, y, and z*

Do this by applying the principle of dimensional homogeneity to Eq. *b*. Thus, $0 = z$; $0 = x + y - 2z + 1$; $0 = -2x + z - 1$. Solving these simultaneous equations yields $x = -1/2$; $y = -1/2$; $z = 0$.

7. *Substitute these values in Eq. a*

Thus, $\pi_1 = g^{-1/2} D^{-1/2} V$, or $\pi_1 = V/(gD)^{1/2}$.

8. *Follow the same procedure for the remaining nonrepeating quantities*

Select ρ_w and ρ_a in turn as the nonrepeating quantities. Follow the same procedure as before to obtain the following dimensionless parameters: $\pi_2 = \rho_w(gD^3)^{1/2}/\mu_a$, and $\pi_3 = \rho_a(gD^3)^{1/2}/\mu_a = (gD^3)^{1/2}/v_a$, where v_a = kinematic viscosity of air.

HYDRAULIC SIMILARITY AND CONSTRUCTION OF MODELS

A dam discharges 36,000 ft^3/s (1,019,236.7 L/s) of water, and a hydraulic pump occurs on the apron. The power loss resulting from this jump is to be determined by constructing a geometrically similar model having a scale of 1:12. (*a*) Determine the required discharge in the model. (*b*) Determine the power loss on the dam if the power loss on the model is found to be 0.18 hp (0.134 kW).

Calculation Procedure:

1. *Determine the value of Q_m*

Two systems are termed similar if their corresponding variables have a constant ratio. A hydraulic model and its prototype must possess three forms of similarity: geometric, or similarity of shape; kinematic, or similarity of motion; and dynamic, or similarity of forces.

In the present instance, the ratio associated with the geometric similarity is given, i.e., the ratio of a linear dimension in the model to the corresponding linear dimension in the prototype. Let r_g denote this ratio, and let subscripts *m* and *p* refer to the model and prototype, respectively.

Apply Eq. 17*a* to evaluate Q_m. Or $Q = C_1bh^{1.5}$, where C_1 is a constant. Then $Q_m/Q_p = (b_m/h_p)(b_m/h_p)^{1.5}$. But $b_m/b_p = h_m/h_p = r_g$; therefore, $Q_m/Q_p = r_g^{2.5} = (1/12)^{2.5} = 1/499$; $Q_m = 36{,}000/499 = 72$ ft^3/s (2038.5 L/s).

2. *Evaluate the power loss on the dam*

Apply Eq. 16 to evaluate the power loss on the dam. Thus, $hp = C_2Qh$, where C_2 is a constant. Then $hp_p/hp_m = (Q_p/Q_m)(h_p/h_m)$. But $Q_p/Q_m = (1/r_g)^{2.5}$, and $h_p/h_m = 1/r_g$; therefore, $hp_p/hp_m = (1/r_g)^{3.5} = 12^{3.5} = 5990$. Hence, $hp_p = 5990(0.18) = 1078$ hp (803.86 kW).

PART 2

PUMP OPERATING MODES, AFFINITY LAWS, SPEED, AND HEAD

SERIES PUMP INSTALLATION ANALYSIS

A new plant requires a system pumping capability of 45 gal/min (2.84 L/s) at a 26-ft (7.9-m) head. The pump characteristic curves for the tentatively selected floor-mounted units are shown in Fig. 15; one operating pump and one standby pump, each 0.75 hp (0.56 kW) are being considered. Can energy be conserved, and how much, with some other pumping arrangement?

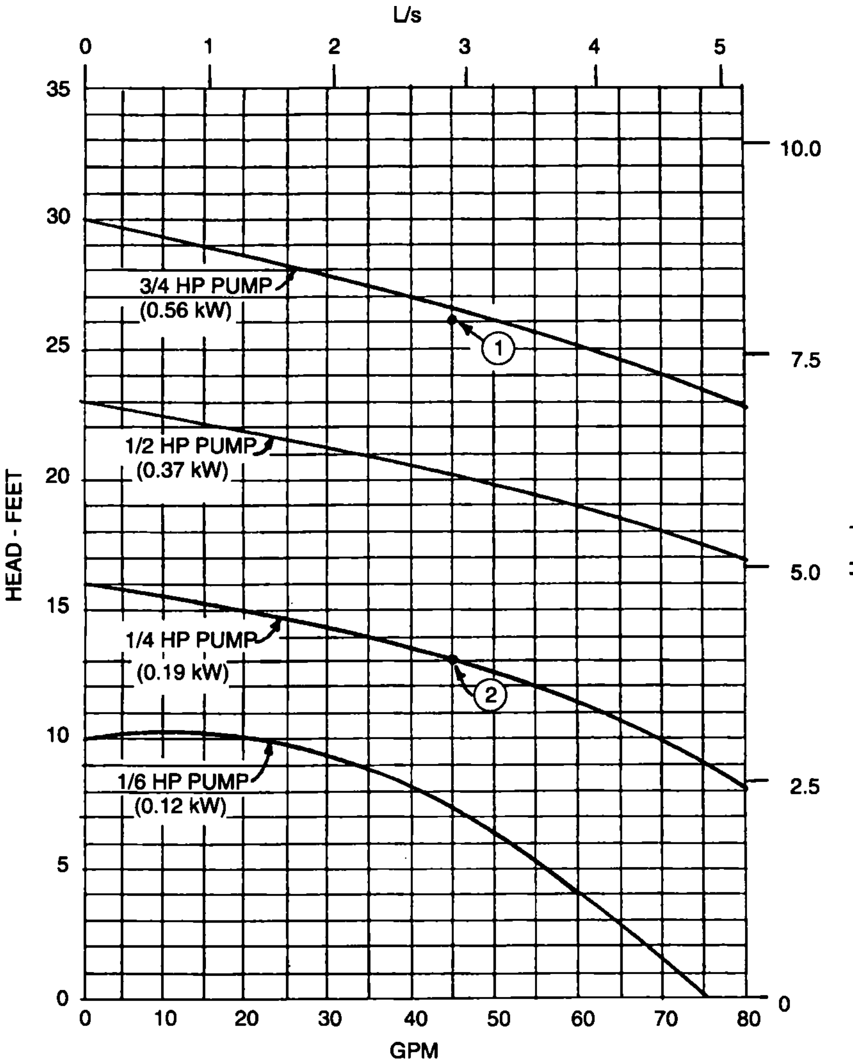


FIGURE 15. Pump characteristic curves for use in series installation.

Calculation Procedure:

1. *Plot the characteristic curves for the pumps being considered*

Figure 15 shows the characteristic curves for the proposed pumps. Point 1 in Fig. 15 is the proposed operating head and flow rate. An alternative pump choice is shown at Point 2 in Fig. 15. If two of the smaller pumps requiring only 0.25 hp (0.19 kW) each are placed in series, they can generate the required 26-ft (7.9-m) head.

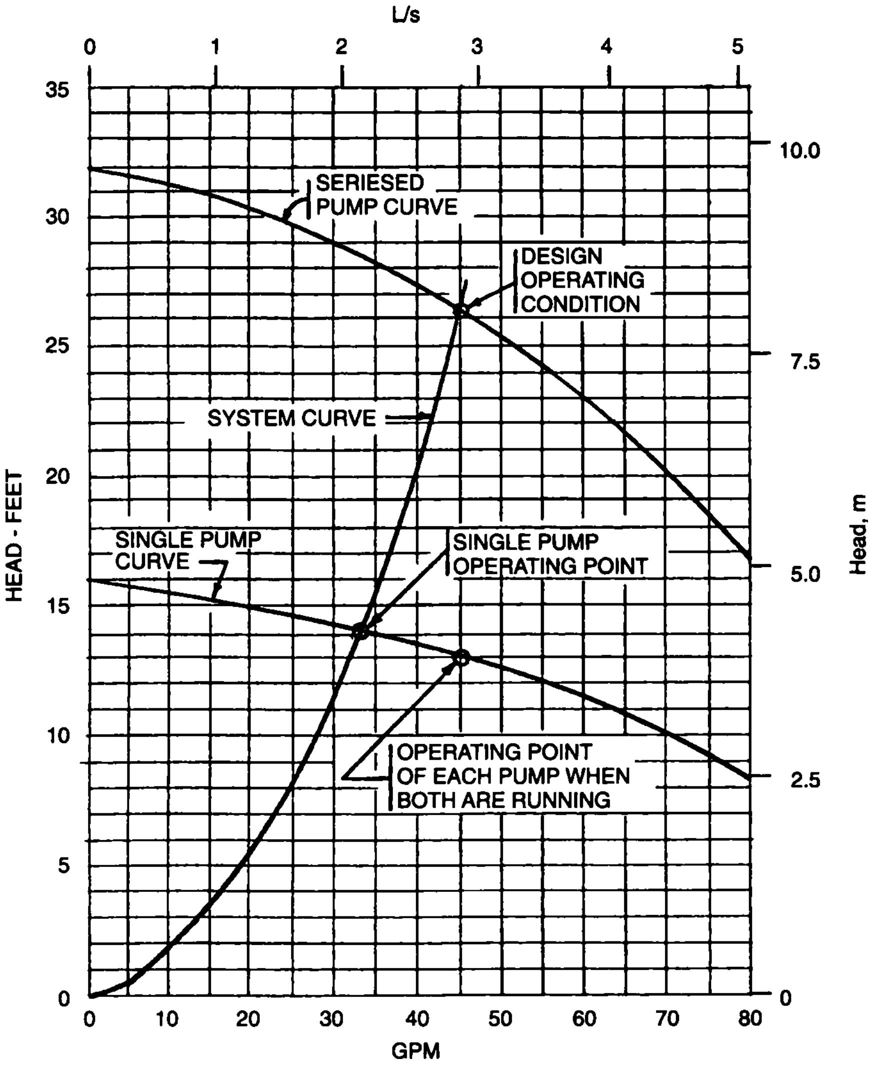


FIGURE 16. Seriesed-pump characteristic and system-head curves.

2. *Analyze the proposed pumps*

To analyze properly the proposal, a new set of curves, Fig. 16, is required. For the proposed series pumping application, it is necessary to establish a *seriesed pump curve.* This is a plot of the head and flow rate (capacity) which exists when both pumps are running in series. To construct this curve, double the single-pump head values at any given flow rate.

Next, to determine accurately the flow a single pump can deliver, plot the system-head curve using the same method fully described in the next calculation procedure. This curve is also plotted on Fig. 16.

Plot the point of operation for each pump on the seriesed curve, Fig. 16. The point of operation of each pump is on the single-pump curve when both pumps are operating. Each pump supplies half the total required head.

When a single pump is running, the point of operation will be at the intersection of the system-head curve and the single-pump characteristic curve, Fig. 16. At this point both the flow and the hp (kW) input of the single pump decrease. Series pumping, Fig. 16, requires the input motor hp (kW) for both pumps; this is the point of maximum power input.

3. *Compute the possible savings*
If the system requires a constant flow of 45 gal/min (2.84 L/s) at 26-ft (7.9-m) head the two-pump series installation saves (0.75 hp − 2 × 0.25 hp) = 0.25 hp (0.19 kW) for every hour the pumps run. For every 1000 hours of operation, the system saves 190 kWh. Since 2000 hours are generally equal to one shift of operation per year, the saving is 380 kWh per shift per year.

If the load is frequently less than peak, one-pump operation delivers 32.5 gal/min (2.1 L/s). This value, which is some 72 percent of full load, corresponds to doubling the saving.

Related Calculations. Series operation of pumps can be used in a variety of designs for industrial, commercial, residential, chemical, power, marine, and similar plants. A series connection of pumps is especially suitable when full-load demand is small; i.e., just a few hours a week, month, or year. With such a demand, one pump can serve the plant's needs most of the time, thereby reducing the power bill. When full-load operation is required, the second pump is started. If there is a need for maintenance of the first pump, the second unit is available for service.

This procedure is the work of Jerome F. Mueller, P.E., of Mueller Engineering Corp.

PARALLEL PUMPING ECONOMICS

A system requires a flow of 80 gal/min (7.4 L/s) of 200°F (92.5°C) water at a 20°F (36°C) temperature drop and a 13-ft (3.96-m) system head. The required system flow can be handled by two pumps, one an operating unit and one a spare unit. Each pump will have an 0.5-hp (0.37-kW) drive motor. Could there be any appreciable energy saving using some other arrangement? The system requires 50 hours of constant pump operation and 40 hours of partial pump operation per week.

Calculation Procedure:

1. *Plot characteristic curves for the proposed system*
Figure 17 shows the proposed pump selection. Looking at the values of the pump head and capacity in Fig. 17, it can be seen that if the peak load of 80 gal/min (7.4 L/s) were carried by two pumps, then each would have to pump only 40 gal/min (3.7 L/s) in a parallel arrangement.

2. *Plot a characteristic curve for the pumps in parallel*
Construct the paralleled-pump curve by doubling the flow of a single pump at any given head, using data from the pump manufacturer. At 13-ft head (3.96-m) one pump produces 40 gal/min (3.7 L/s); two pumps 80 gal/min (7.4 L/s). The resulting curve is shown in Fig. 18.

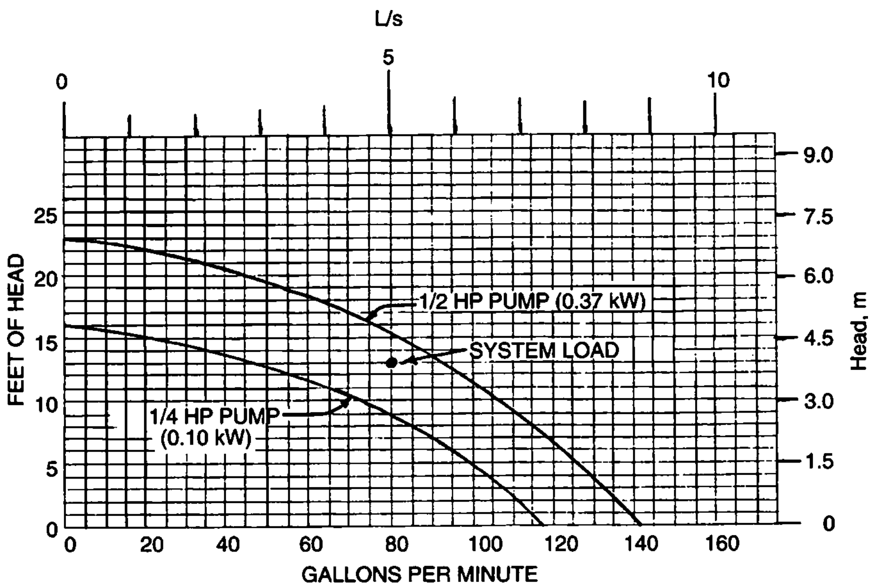


FIGURE 17. Typical pump characteristic curves.

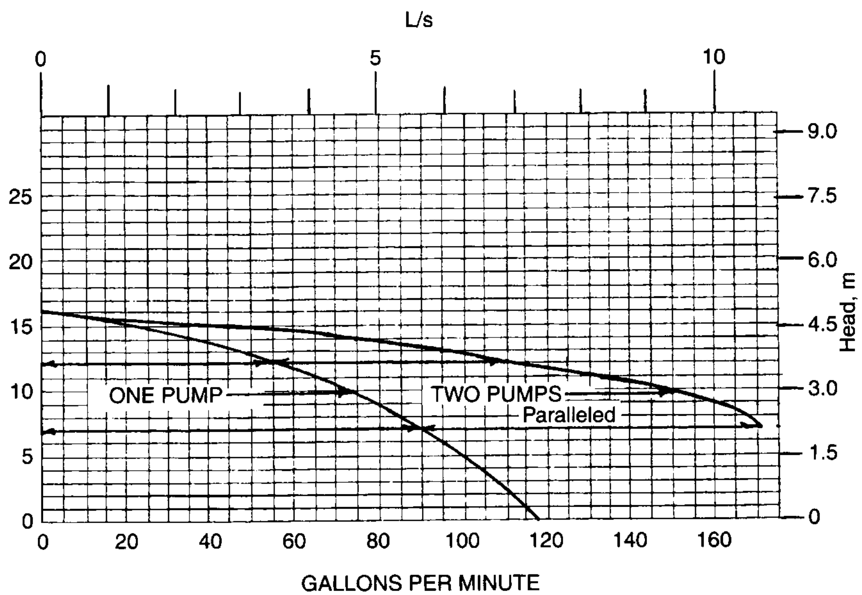


FIGURE 18. Single- and dual-parallel pump characteristic curves.

The load for this system could be divided among three, four, or more pumps, if desired. To achieve the best results, the number of pumps chosen should be based on achieving the proper head and capacity requirements in the system.

3. *Construct a system-head curve*

Based on the known flow rate, 80 gal/min (7.4 L/s) at 13-ft (3.96-m) head, a system-head curve can be constructed using the fact that pumping head varies as the square of the change in flow, or $Q_2/Q_1 = H_2/H_1$, where Q_1 = known design flow, gal/min (L/s); Q_2 = selected flow, gal/min (L/s); H_1 = known design head, ft (m); H_2 = resultant head related to selected flow rate, gal/min (L/s).

Figure 19 shows the plotted system-head curve. Once the system-head curve is plotted, draw the single-pump curve from Fig. 17 on Fig. 19, and the paralleled-pump curve from Fig. 18. Connect the different pertinent points of concern with dashed lines, Fig. 19.

The point of crossing of the two-pump curve and the system-head curve is at the required value of 80 gal/min (7.4 L/s) and 13-ft (3.96-m) head because it was so planned. But the point of crossing of the system-head curve and the single-pump curve is of particular interest.

The single pump, instead of delivering 40 gal/min (7.4 L/s) at 13-ft (3.96-m) head will deliver, as shown by the intersection of the curves in Fig. 19, 72 gal/min (6.67 L/s) at 10-ft (3.05-m) head. Thus, the single pump can effectively be a standby for 90 percent of the required capacity at a power input of 0.5 hp (0.37 kW). Much of the time in heating and air conditioning, and frequently in industrial processes, the system load is 90 percent, or less.

4. *Determine the single-pump horsepower input*

In the installation here, the pumps are the inline type with non-overload motors. For larger flow rates, the pumps chosen would be floor-mounted units providing a variety of

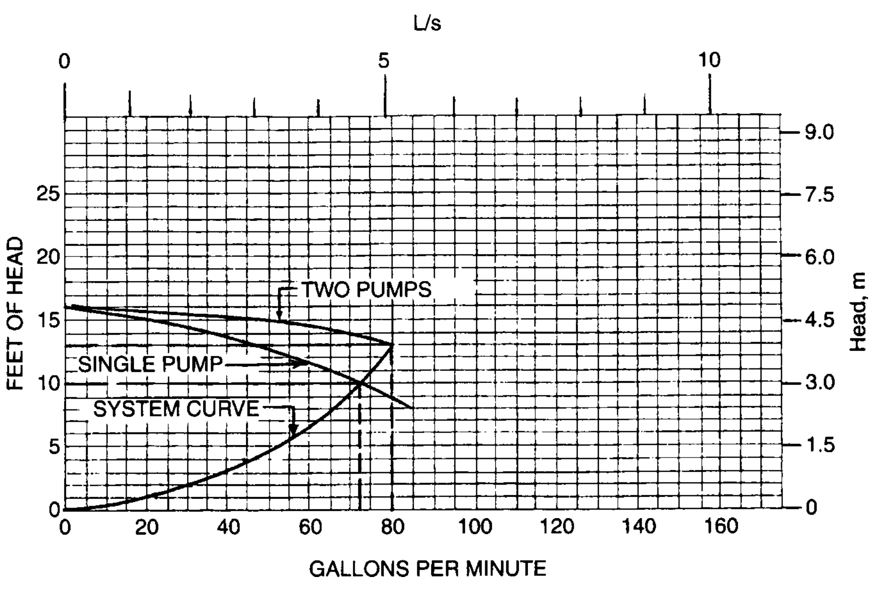


FIGURE 19. System-head curve for parallel pumping.

horsepower (kW) and flow curves. The horsepower (kW) for—say a 200-gal/min (18.6 L/s) flow rate would be about half of a 400-gal/min (37.2 L/s) flow rate.

If a pump were suddenly given a 300-gal/min (27.9 L/s) flow-rate demand at its crossing point on a larger system-head curve, the hp required might be excessive. Hence, the pump drive motor must be chosen carefully so that the power required does not exceed the motor's rating. The power input required by any pump can be obtained from the pump characteristic curve for the unit being considered. Such curves are available free of charge from the pump manufacturer.

The pump operating point is at the intersection of the pump characteristic curve and the system-head curve in conformance with the first law of thermodynamics, which states that the energy put into the system must exactly match the energy used by the system. The intersection of the pump characteristic curve and the system-head curve is the only point that fulfills this basic law.

There is no practical limit for pumps in parallel. Careful analysis of the system-head curve versus the pump characteristic curves provided by the pump manufacturer will frequently reveal cases where the system load point may be beyond the desired pump curve. The first cost of two or three smaller pumps is frequently no greater than for one large pump. Hence, smaller pumps in parallel may be more desirable than a single large pump, from both the economic and reliability standpoints.

One frequently overlooked design consideration in piping for pumps is shown in Fig. 20. This is the location of the check valve to prevent reverse-flow pumping. Figure 20 shows the proper location for this simple valve.

5. *Compute the energy saving possible*

Since one pump can carry the fluid flow load about 90 percent of the time, and this same percentage holds for the design conditions, the saving in energy is $0.9 \times (0.5 \text{ kW} - .25 \text{ kW}) \times 90$ h per week = 20.25 kWh/week. (In this computation we used the assumption that 1 hp = 1 kW.) The annual savings would be 52 weeks × 20.25 kW/week = 1053 kWh/yr. If electricity costs 5 cents per kWh, the annual saving is \$0.05 × 1053 = \$52.65/yr.

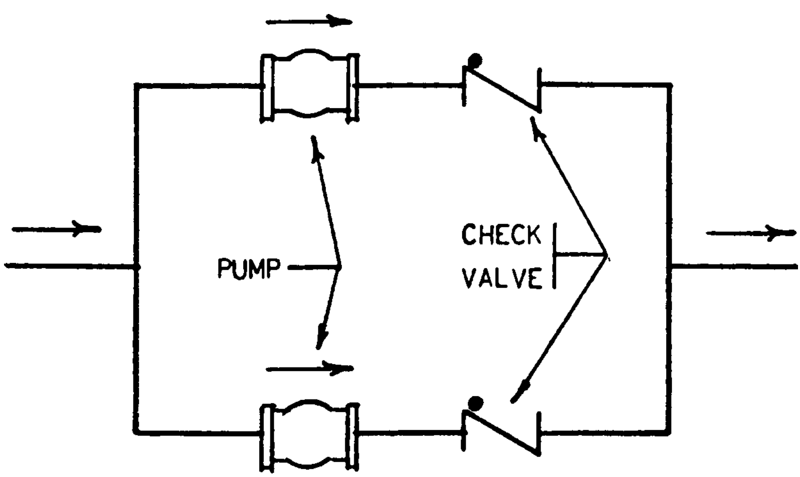


FIGURE 20. Check valve locations to prevent reverse flow.

While a saving of some \$52 per year may seem small, such a saving can become much more if: (1) larger pumps using higher horsepower (kW) motors are used; (2) several hundred pumps are used in the system; (3) the operating time is longer—168 hours per week in some systems. If any, or all, of these conditions prevail, the savings can be substantial.

Related Calculations. This procedure can be used for pumps in a variety of applications: industrial, commercial, residential, medical, recreational, and similar systems. When analyzing any system the designer should be careful to consider all the available options so the best one is found.

This procedure is the work of Jerome F. Mueller, P.E., of Mueller Engineering Corp.

SIMILARITY OR AFFINITY LAWS FOR CENTRIFUGAL PUMPS

A centrifugal pump designed for an 1800-r/min operation and a head of 200 ft (60.9 m) has a capacity of 3000 gal/min (189.3 L/s) with a power input of 175 hp (130.6 kW). What effect will a speed reduction to 1200 r/min have on the head, capacity, and power input of the pump? What will be the change in these variables if the impeller diameter is reduced from 12 to 10 in. (304.8 to 254 mm) while the speed is held constant at 1800 r/min?

Calculation Procedure:

1. *Compute the effect of a change in pump speed*

For any centrifugal pump in which the effects of fluid viscosity are negligible, or are neglected, the similarity or affinity laws can be used to determine the effect of a speed, power, or head change. For a *constant impeller diameter,* the laws are $Q_1/Q_2 = N_1/N_2$; $H_1/H_2 = (N_1/N_2)^2$; $P_1/P_2 = (N_1/N_2)^3$. For a *constant speed,* $Q_1/Q_2 = D_1/D_2$; $H_1/H_2 = (D_1/D_2)^2$; $P_1/P_2 = (D_1/D_2)^3$. In both sets of laws, Q = capacity, gal/min; N = impeller rpm; D = impeller diameter, in.; H = total head, ft of liquid; P = bhp input. The subscripts 1 and 2 refer to the initial and changed conditions, respectively.

For this pump, with a constant impeller diameter, $Q_1/Q_2 = N_1/N_2$; $3000/Q_2 = 1800/1200$; $Q_2 = 2000$ gal/min (126.2 L/s). And, $H_1/H_2 = (N_1/N_2)^2 = 200/H_2 = (1800/1200)^2$; $H_2 = 88.9$ ft (27.1 m). Also, $P_1/P_2 = (N_1/N_2)^3 = 175/P_2 = (1800/1200)^3$; $P_2 = 51.8$ bhp (38.6 kW).

2. *Compute the effect of a change in impeller diameter*

With the speed constant, use the second set of laws. Or, for this pump, $Q_1/Q_2 = D_1/D_2$; $3000/Q_2 = 12/10$; $Q_2 = 2500$ gal/min (157.7 L/s). And $H_1/H_2 = (D_1/D_2)^2$; $200/H_2 = (12/10)^2$; $H_2 = 138.8$ ft (42.3 m). Also, $P_1/P_2 = (D_1/D_2)^3$; $175/P_2 = (12/10)^3$; $P_2 = 101.2$ bhp (75.5 kW).

Related Calculations. Use the similarity laws to extend or change the data obtained from centrifugal pump characteristic curves. These laws are also useful in field calculations when the pump head, capacity, speed, or impeller diameter is changed.

The similarity laws are most accurate when the efficiency of the pump remains nearly constant. Results obtained when the laws are applied to a pump having a constant impeller diameter are somewhat more accurate than for a pump at constant speed with a changed impeller diameter. The latter laws are more accurate when applied to pumps having a low specific speed.

If the similarity laws are applied to a pump whose impeller diameter is increased, be certain to consider the effect of the higher velocity in the pump suction line. Use the similarity laws for any liquid whose viscosity remains constant during passage through the pump. However, the accuracy of the similarity laws decreases as the liquid viscosity increases.

SIMILARITY OR AFFINITY LAWS IN CENTRIFUGAL PUMP SELECTION

A test-model pump delivers, at its best efficiency point, 500 gal/min (31.6 L/s) at a 350-ft (106.7-m) head with a required net positive suction head (NPSH) of 10 ft (3 m) and a power input of 55 hp (41 kW) at 3500 r/min, when a 10.5-in. (266.7-mm) diameter impeller is used. Determine the performance of the model at 1750 r/min. What is the performance of a full-scale prototype pump with a 20-in. (50.4-cm) impeller operating at 1170 r/min? What are the specific speeds and the suction specific speeds of the test-model and prototype pumps?

Calculation Procedure:

1. *Compute the pump performance at the new speed*

The similarity or affinity laws can be stated in general terms, with subscripts *p* and *m* for prototype and model, respectively, as $Q_p = K_d^3 N_n Q_m$; $H_p = K_d^2 K_n^2 H_m$; $\text{NPSH}_p = K_2^2 K_n^2 \text{NPSH}_m$; $P_p = K_d^5 K_n^5 P_m$, where K_d = size factor = prototype dimension/model dimension. The usual dimension used for the size factor is the impeller diameter. Both dimensions should be in the same units of measure. Also, K_n = (prototype speed, r/min)/(model speed, r/min). Other symbols are the same as in the previous calculation procedure.

When the model speed is reduced from 3500 to 1750 r/min, the pump dimensions remain the same and $K_d = 1.0$; $K_n = 1750/3500 = 0.5$. Then $Q = (1.0)(0.5)(500) =$ 250 r/min; $H = (1.0)^2(0.5)^2(350) = 87.5$ ft (26.7 m); $\text{NPSH} = (1.0)^2(0.5)^2(10) = 2.5$ ft (0.76 m); $P = (1.0)^5(0.5)^3(55) = 6.9$ hp (5.2 kW). In this computation, the subscripts were omitted from the equations because the same pump, the test model, was being considered.

2. *Compute performance of the prototype pump*

First, K_d and K_n must be found: $K_d = 20/10.5 = 1.905$; $K_n = 1170/3500 = 0.335$. Then $Q_p = (1.905)^3(0.335)(500) = 1158$ gal/min (73.1 L/s); $H_p = (1.905)^2(0.335)^2(350) =$ 142.5 ft (43.4 m); $\text{NPSH}_p = (1.905)^2(0.335)^2(10) = 4.06$ ft (1.24 m); $P_p = (1.905)^5(0.335)^3(55) = 51.8$ hp (38.6 kW).

3. *Compute the specific speed and suction specific speed*

The specific speed or, as Horwitz[1] says, "more correctly, discharge specific speed," is $N_s = N(Q)^{0.5}/(H)^{0.75}$, while the suction specific speed $S = N(Q)^{0.5}/(\text{NPSH})^{0.75}$, where all values are taken at the best efficiency point of the pump.

For the model, $N_s = 3500(500)^{0.5}/(350)^{0.75} = 965$; $S = 3500(500)^{0.5}/(10)^{0.75} =$ 13,900. For the prototype, $N_s = 1170(1158)^{0.5}/(142.5)^{0.75} = 965$; $S = 1170(1156)^{0.5}/(4.06)^{0.75} = 13{,}900$. The specific speed and suction specific speed of the model and prototype are equal because these units are geometrically similar or homologous pumps and both speeds are mathematically derived from the similarity laws.

Related Calculations. Use the procedure given here for any type of centrifugal pump where the similarity laws apply. When the term *model* is used, it can apply to a production test pump or to a standard unit ready for installation. The procedure presented here is the work of R. P. Horwitz, as reported in *Power* magazine.[1]

[1]R. P. Horwitz, "Affinity Laws and Specific Speed Can Simplify Centrifugal Pump Selection," *Power,* November 1964.

SPECIFIC SPEED CONSIDERATIONS IN CENTRIFUGAL PUMP SELECTION

What is the upper limit of specific speed and capacity of a 1750-r/min single-stage double-suction centrifugal pump having a shaft that passes through the impeller eye if it handles clear water at 85°F (29.4°C) at sea level at a total head of 280 ft (85.3 m) with a 10-ft (3-m) suction lift? What is the efficiency of the pump and its approximate impeller shape?

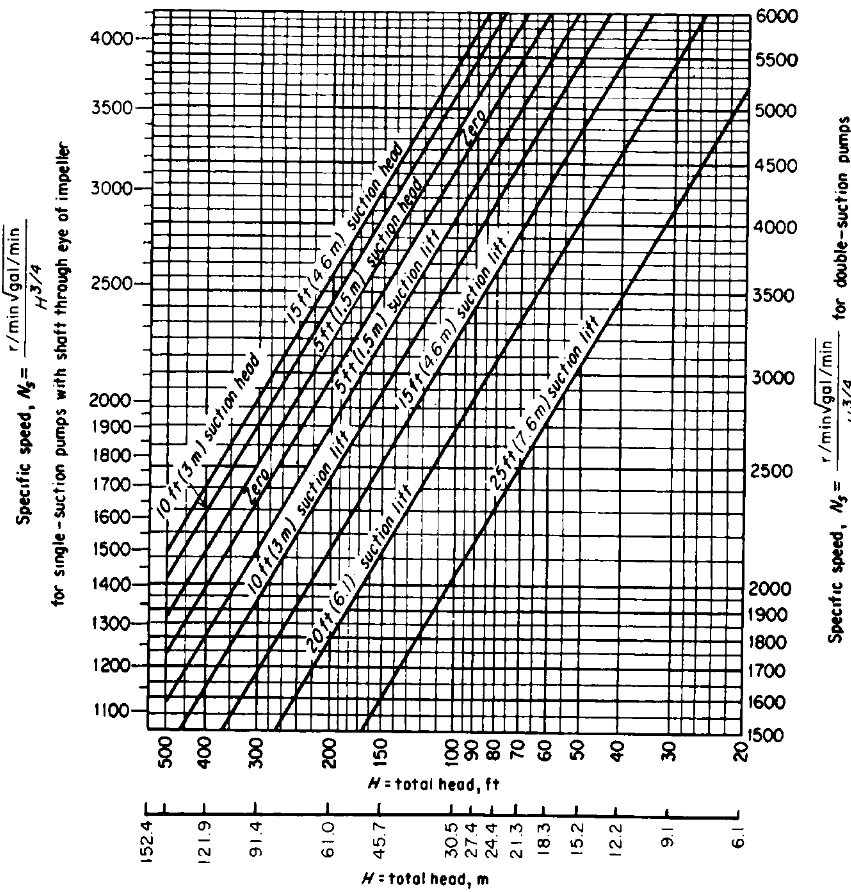


FIGURE 21. Upper limits of specific speeds of single-stage, single- and double-suction centrifugal pumps handling clear water at 85°F (29.4°C) at sea level. (*Hydraulic Institute.*)

Calculation Procedure:

1. *Determine the upper limit of specific speed*

Use the Hydraulic Institute upper specific-speed curve, Fig. 21, for centrifugal pumps or a similar curve, Fig. 22, for mixed- and axial-flow pumps. Enter Fig. 1 at the bottom at 280-ft (85.3-m) total head, and project vertically upward until the 10-ft (3-m) suction-lift curve is intersected. From here, project horizontally to the right to read the specific speed $N_s = 2000$. Figure 2 is used in a similar manner.

2. *Compute the maximum pump capacity*

For any centrifugal, mixed- or axial-flow pump, $N_S = (gpm)^{0.5}(rpm)/H_t^{0.75}$, where H_t = total head on the pump, ft of liquid. Solving for the maximum capacity, we get $gpm = (N_S H_t^{0.75}/rpm)^2 = (2000 \times 280^{0.75}/1750)^2 = 6040$ gal/min (381.1 L/s).

3. *Determine the pump efficiency and impeller shape*

Figure 23 shows the general relation between impeller shape, specific speed, pump capacity, efficiency, and characteristic curves. At $N_S = 2000$, efficiency = 87 percent. The impeller, as shown in Fig. 23, is moderately short and has a relatively large discharge area. A cross section of the impeller appears directly under the N_S = 2000 ordinate.

Related Calculations. Use the method given here for any type of pump whose variables are included in the Hydraulic Institute curves, Figs. 21 and 22, and in similar curves available from the same source. *Operating specific speed,* computed as above, is sometimes plotted on the performance curve of a centrifugal pump so that the characteristics of the unit can be better understood. *Type specific speed* is the operating specific

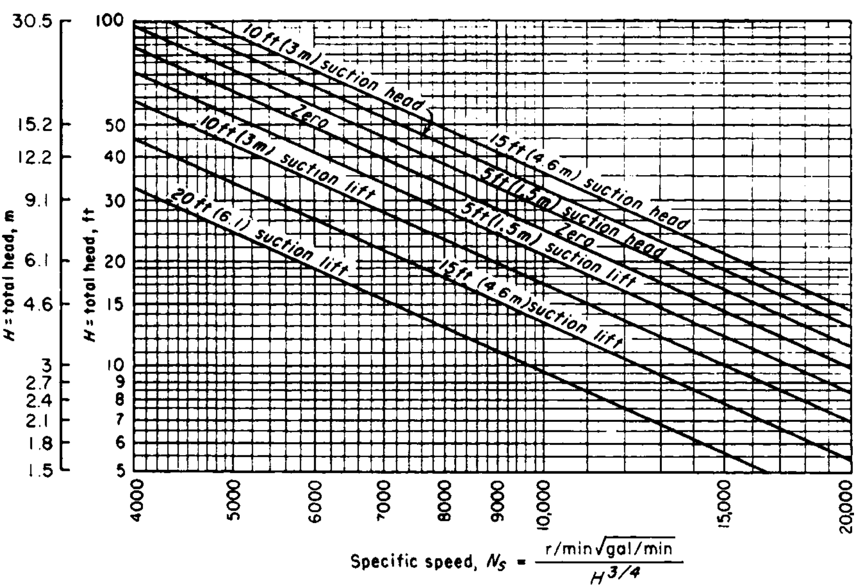


FIGURE 22. Upper limits of specific speeds of single-suction mixed-flow and axial-flow pumps. (*Hydraulic Institute.*)

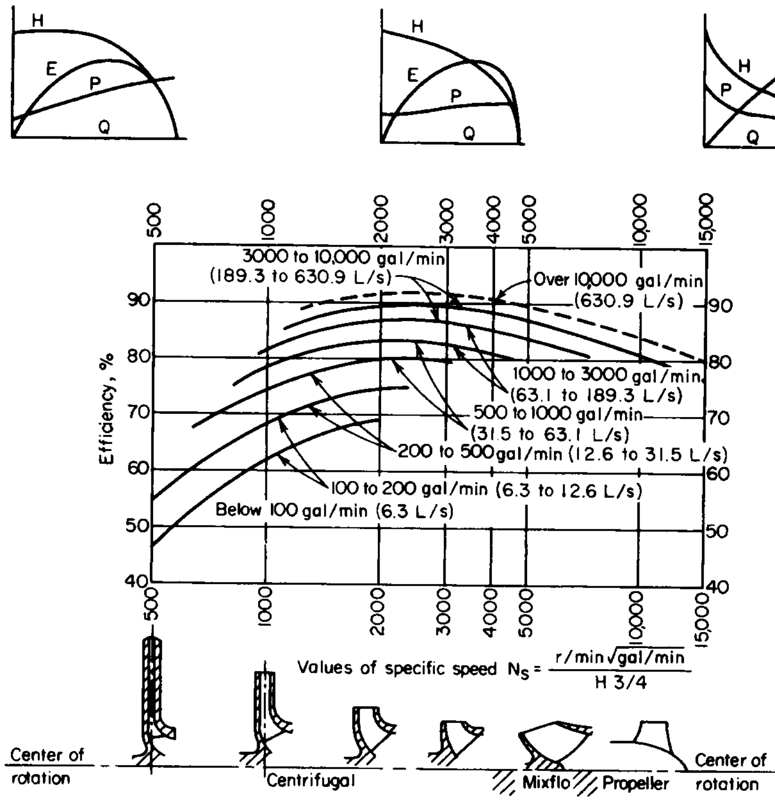


FIGURE 23. Approximate relative impeller shapes and efficiency variations for various specific speeds of centrifugal pumps. (*Worthington Corporation.*)

speed giving maximum efficiency for a given pump and is a number used to identify a pump. Specific speed is important in cavitation and suction-lift studies. The Hydraulic Institute curves, Figs. 21 and 22, give upper limits of speed, head, capacity and suction lift for cavitation-free operation. When making actual pump analyses, be certain to use the curves (Figs. 21 and 22) in the latest edition of the *Standards of the Hydraulic Institute.*

SELECTING THE BEST OPERATING SPEED FOR A CENTRIFUGAL PUMP

A single-suction centrifugal pump is driven by a 60-Hz ac motor. The pump delivers 10,000 gal/min (630.9 L/s) of water at a 100-ft (30.5-m) head. The available net positive suction head = 32 ft (9.7 m) of water. What is the best operating speed for this pump if the pump operates at its best efficiency point?

TABLE 1. Pump Types Listed by Specific Speed*

Specific speed range	Type of pump
Below 2,000	Volute, diffuser
2,000–5,000	Turbine
4,000–10,000	Mixed-flow
9,000–15,000	Axial-flow

*Peerless Pump Division, FMC Corporation.

Calculation Procedure:

1. *Determine the specific speed and suction specific speed*

Ac motors can operate at a variety of speeds, depending on the number of poles. Assume that the motor driving this pump might operate at 870, 1160, 1750, or 3500 r/min. Compute the specific speed $N_S = N(Q)^{0.5}/(H)^{0.75} = N(10{,}000)^{0.5}/(100)^{0.75} = 3.14N$ and the suction specific speed $S = N(Q)^{0.5}/(\text{NPSH})^{0.75} = N(10{,}000)^{0.5}/(32)^{0.75} = 7.43N$ for each of the assumed speeds. Tabulate the results as follows:

Operating speed, r/min	Required specific speed	Required suction specific speed
870	2,740	6,460
1,160	3,640	8,620
1,750	5,500	13,000
3,500	11,000	26,000

2. *Choose the best speed for the pump*

Analyze the specific speed and suction specific speed at each of the various operating speeds, using the data in Tables 1 and 2. These tables show that at 870 and 1160 r/min, the suction specific-speed rating is poor. At 1750 r/min, the suction specific-speed rating is excellent, and a turbine or mixed-flow type pump will be suitable. Operation at 3500 r/min is unfeasible because a suction specific speed of 26,000 is beyond the range of conventional pumps.

Related Calculations. Use this procedure for any type of centrifugal pump handling water for plant services, cooling, process, fire protection, and similar requirements.

TABLE 2. Suction Specific-Speed Ratings*

Single-suction pump	Double-suction pump	Rating
Above 11,000	Above 14,000	Excellent
9,000–11,000	11,000–14,000	Good
7,000–9,000	9,000–11,000	Average
5,000–7,000	7,000–9,000	Poor
Below 5,000	Below 7,000	Very poor

*Peerless Pump Division, FMC Corporation.

This procedure is the work of R. P. Horwitz, Hydrodynamics Division, Peerless Pump, FMC Corporation, as reported in *Power* magazine.

TOTAL HEAD ON A PUMP HANDLING VAPOR-FREE LIQUID

Sketch three typical pump piping arrangements with static suction lift and submerged, free, and varying discharge head. Prepare similar sketches for the same pump with static suction head. Label the various heads. Compute the total head on each pump if the elevations are as shown in Fig. 24 and the pump discharges a maximum of 2000 gal/min (126.2 L/s) of water through 8-in. (203.2-mm) schedule 40 pipe. What hp is required to drive the

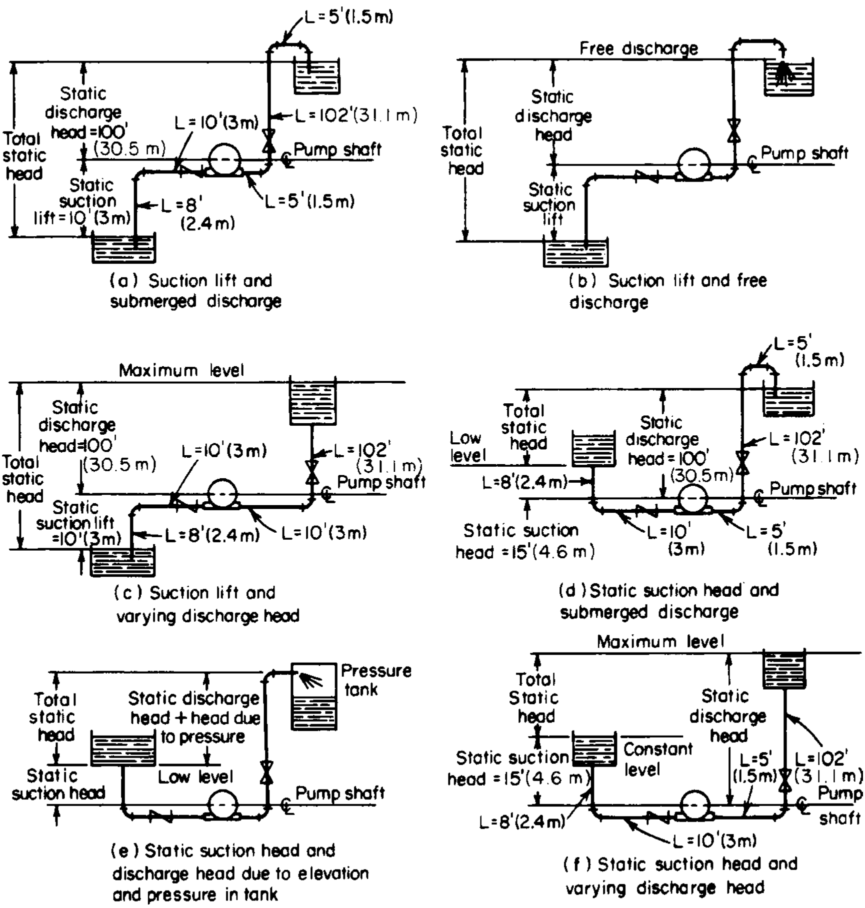


FIGURE 24. Typical pump suction and discharge piping arrangements.

pump? A swing check valve is used on the pump suction line and a gate valve on the discharge line.

Calculation Procedure:

1. *Sketch the possible piping arrangements*

Figure 24 shows the six possible piping arrangements for the stated conditions of the installation. Label the total static head, i.e., the *vertical* distance from the surface of the source of the liquid supply to the free surface of the liquid in the discharge receiver, or to the point of free discharge from the discharge pipe. When both the suction and discharge surfaces are open to the atmosphere, the total static head equals the vertical difference in elevation. Use the free-surface elevations that cause the maximum suction lift and discharge head, i.e., the *lowest* possible level in the supply tank and the *highest* possible level in the discharge tank or pipe. When the supply source is *below* the pump centerline, the vertical distance is called the *static suction lift*; with the supply *above* the pump centerline, the vertical distance is called *static suction head.* With variable static suction head, use the lowest liquid level in the supply tank when computing total static head. Label the diagrams as shown in Fig. 24.

2. *Compute the total static head on the pump*

The total static head H_{ts} ft = static suction lift, h_{sl} ft + static discharge head h_{sd} ft, where the pump has a suction lift, *s* in Fig. 24*a*, *b*, and *c*. In these installations, $H_{ts} = 10 + 100 = 110$ ft (33.5 m). Note that the static discharge head is computed between the pump centerline and the water level with an underwater discharge, Fig. 24*a*; to the pipe outlet with a free discharge, Fig. 24*b*; and to the maximum water level in the discharge tank, Fig. 24*c*. When a pump is discharging into a closed compression tank, the total discharge head equals the static discharge head plus the head equivalent, ft of liquid, of the internal pressure in the tank, or 2.31 × tank pressure, lb/sq.in.

Where the pump has a static suction head, as in Fig. 24*d*, *e*, and *f*, the total static head H_{ts} ft = h_{sd} − static suction head h_{sh} ft. In these installations, $H_t = 100 - 15 = 85$ ft (25.9 m).

The total static head, as computed above, refers to the head on the pump without liquid flow. To determine the total head on the pump, the friction losses in the piping system during liquid flow must be also determined.

3. *Compute the piping friction losses*

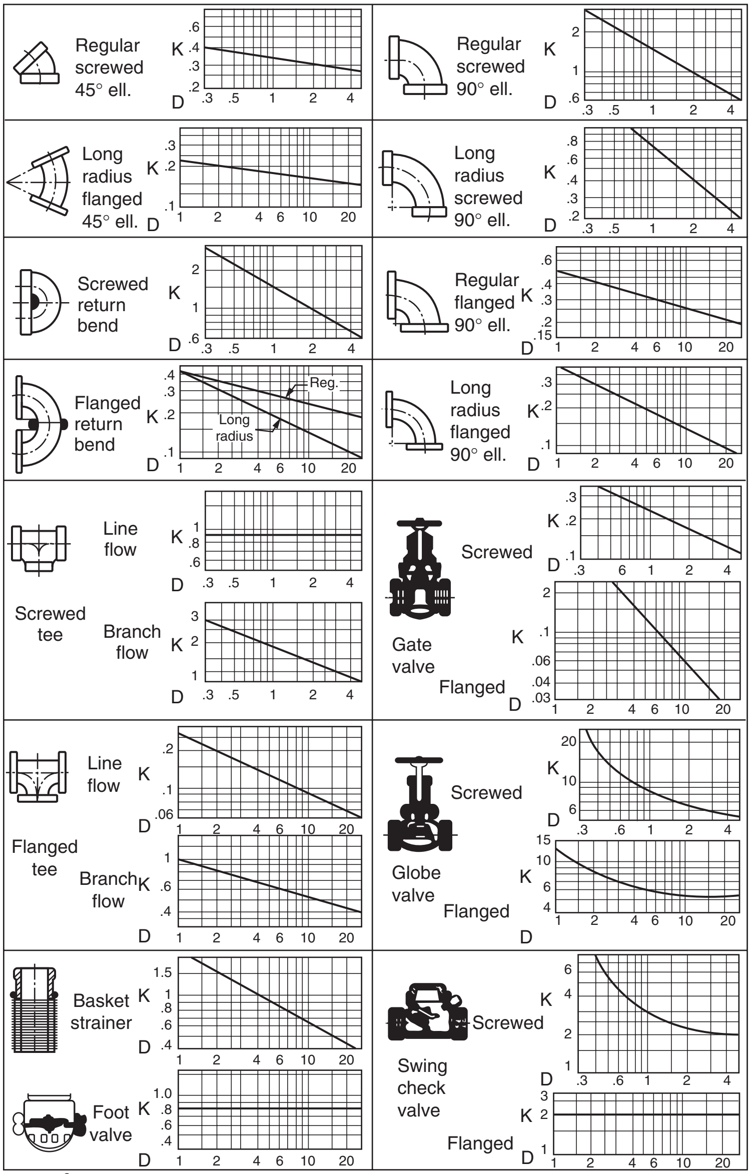
Mark the length of each piece of straight pipe on the piping drawing. Thus, in Fig. 10*a*, the total length of straight pipe L_t ft = 8 + 10 + 5 + 102 + 5 = 130 ft (39.6 m), if we start at the suction tank and add each length until the discharge tank is reached. To the total length of straight pipe must be added the *equivalent* length of the pipe fittings. In Fig. 10*a* there are four long-radius elbows, one swing check valve, and one gate valve. In addition, there is a minor head loss at the pipe inlet and at the pipe outlet.

The equivalent length of one 8-in. (203.2-mm) long-radius elbow is 14 ft (4.3 m) of pipe, from Table 3. Since the pipe contains four elbows, the total equivalent length = 4(14) = 56 ft (17.1 m) of straight pipe. The open gate valve has an equivalent resistance of 4.5 ft (1.4 m); and the open swing check valve has an equivalent resistance of 53 ft (16.2 m).

The entrance loss h_e ft, assuming a basket-type strainer is used at the suction-pipe inlet, is h_e ft = $Kv^2/2g$, where K = a constant from Fig. 5; v = liquid velocity, ft/s; $g = 32.2$ ft/s^2 (980.67 cm/s^2). The exit loss occurs when the liquid passes through a sudden enlargement, as from a pipe to a tank. Where the area of the tank is large, causing a final velocity that is zero, $h_{ex} = v^2/2g$.

TABLE 3. Resistance of Fittings and Valves (length of straight pipe giving equivalent resistance)

Pipe size		Standard ell		Medium-radius ell		Long-radius ell		45° Ell		Tee		Gate valve, open		Globe valve, open		Swing check, open	
in.	mm	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m
6	152.4	16	4.9	14	4.3	11	3.4	7.7	2.3	33	10.1	3.5	1.1	160	48.8	40	12.2
8	203.2	21	6.4	18	5.5	14	4.3	10	3.0	43	13.1	4.5	1.4	220	67.0	53	16.2
10	254.0	26	7.9	22	6.7	17	5.2	13	3.9	56	17.1	5.7	1.7	290	88.4	67	20.4
12	304.8	32	9.8	26	7.9	20	6.1	15	4.6	66	20.1	6.7	2.0	340	103.6	80	24.4



$$h = k\frac{v^2}{2g} \text{ feet of fluid}$$

FIGURE 25. Resistance coefficients of pipe fittings. To convert to SI in the equation for h, v^2 would be measured in m/s and feet would be changed to meters. The following values would also be changed from inches to millimeters: 0.3 to 7.6, 0.5 to 12.7, 1 to 25.4, 2 to 50.8, 4 to 101.6, 6 to 152.4, 10 to 254, and 20 to 508. (*Hydraulic Institute*.)

The velocity v ft/s in a pipe = $gpm/2.448d^2$. For this pipe, $v = 2000/[(2.448)(7.98)^2)] = 12.82$ ft/s (3.91 m/s). Then $h_e = 0.74(12.82)^2/[2(23.2)] = 1.89$ ft (0.58 m), and $h_{ex} = (12.82)^2/[(2)(32.2)] = 2.56$ ft (0.78 m). Hence, the total length of the piping system in Fig. 4*a* is 130 + 56 + 4.5 + 53 + 1.89 + 2.56 = 247.95 ft (75.6 m), say 248 ft (75.6 m).

Use a suitable head-loss equation, or Table 4, to compute the head loss for the pipe and fittings. Enter Table 4 at an 8-in. (203.2-mm) pipe size, and project horizontally across to 2000 gal/min (126.2 L/s) and read the head loss as 5.86 ft of water per 100 ft (1.8 m/30.5 m) of pipe.

The total length of pipe and fittings computed above is 248 ft (75.6 m). Then total friction-head loss with a 2000 gal/min (126.2-L/s) flow is H_f ft = (5.86)(248/100) = 14.53 ft (4.5 m).

4. *Compute the total head on the pump*

The total head on the pump $H_t = H_{ts} + H_f$. For the pump in Fig. 24*a*, $H_t = 110 + 14.53 = 124.53$ ft (37.95 m), say 125 ft (38.1 m). The total head on the pump in Fig. 24*b* and *c* would be the same. Some engineers term the total head on a pump the *total dynamic head* to distinguish between static head (no-flow vertical head) and operating head (rated flow through the pump).

The total head on the pumps in Fig. 24*d*, *c*, and *f* is computed in the same way as described above, except that the total static head is less because the pump has a static suction head. That is, the elevation of the liquid on the suction side reduces the total distance through which the pump must discharge liquid; thus the total static head is less. The static suction head is *subtracted* from the static discharge head to determine the total static head on the pump.

5. *Compute the horsepower required to drive the pump*

The brake hp input to a pump $bhp_i = (gpm)(H_t)(s)/3960e$, where s = specific gravity of the liquid handled; e = hydraulic efficiency of the pump, expressed as a decimal. The usual hydraulic efficiency of a centrifugal pump is 60 to 80 percent; reciprocating pumps, 55 to 90 percent; rotary pumps, 50 to 90 percent. For each class of pump, the hydraulic efficiency decreases as the liquid viscosity increases.

Assume that the hydraulic efficiency of the pump in this system is 70 percent and the specific gravity of the liquid handled is 1.0. Then $bhp_i = (2000)(127)(1.0)/(3960)(0.70) = 91.6$ hp (68.4 kW).

TABLE 4. Pipe Friction Loss for Water (wrought-iron or steel schedule 40 pipe in good condition)

Diameter		Flow		Velocity		Velocity head		Friction loss per 100 ft (30.5 m) of pipe	
in.	mm	gal/min	L/s	ft/s	m/s	ft water	m water	ft water	m water
6	152.4	1000	63.1	11.1	3.4	1.92	0.59	6.17	1.88
6	152.4	2000	126.2	22.2	6.8	7.67	2.3	23.8	7.25
6	152.4	4000	252.4	44.4	13.5	30.7	9.4	93.1	28.4
8	203.2	1000	63.1	6.41	1.9	0.639	0.195	1.56	0.475
8	203.2	2000	126.2	12.8	3.9	2.56	0.78	5.86	1.786
8	203.2	4000	252.4	25.7	7.8	10.2	3.1	22.6	6.888
10	254.0	1000	63.1	3.93	1.2	0.240	0.07	0.497	0.151
10	254.0	3000	189.3	11.8	3.6	2.16	0.658	4.00	1.219
10	254.0	5000	315.5	19.6	5.9	5.99	1.82	10.8	3.292

The theoretical or *hydraulic horsepower* $hp_h = (gpm)(H_t)(s)/3960$, or $hp_h = (2000) = (127)(1.0)/3900 = 64.1$ hp (47.8 kW).

Related Calculations. Use this procedure for any liquid—water, oil, chemical, sludge, etc.—whose specific gravity is known. When liquids other than water are being pumped, the specific gravity and viscosity of the liquid, as discussed in later calculation procedures, must be taken into consideration. The procedure given here can be used for any class of pump—centrifugal, rotary, or reciprocating.

Note that Fig. 25 can be used to determine the equivalent length of a variety of pipe fittings. To use Fig. 25, simply substitute the appropriate K value in the relation $h = Kv^2/2g$, where h = equivalent length of straight pipe; other symbols as before.

PUMP SELECTION FOR ANY PUMPING SYSTEM

Give a step-by-step procedure for choosing the class, type, capacity, drive, and materials for a pump that will be used in an industrial pumping system.

Calculation Procedure:

1. *Sketch the proposed piping layout*

Use a single-line diagram, Fig. 26, of the piping system. Base the sketch on the actual job conditions. Show all the piping, fittings, valves, equipment, and other units in the system. Mark the *actual* and *equivalent* pipe length (see the previous calculation procedure) on the sketch. Be certain to include all vertical lifts, sharp bends, sudden enlargements, storage tanks, and similar equipment in the proposed system.

2. *Determine the required capacity of the pump*

The required capacity is the flow rate that must be handled in gal/min, million gal/day, ft^3/s, gal/h, bbl/day, lb/h, acre·ft/day, mil/h, or some similar measure. Obtain the required flow rate from the process conditions, for example, boiler feed rate, cooling-water flow rate, chemical feed rate, etc. The required flow rate for any process unit is usually given by the manufacturer or can be computed by using the calculation procedures given throughout this handbook.

Once the required flow rate is determined, apply a suitable factor of safety. The value of this factor of safety can vary from a low of 5 percent of the required flow to a high of 50 percent or more, depending on the application. Typical safety factors are in the 10 percent range. With flow rates up to 1000 gal/min (63.1 L/s), and in the selection of process pumps, it is common practice to round a computed required flow rate to the next highest round-number capacity. Thus, with a required flow rate of 450 gal/min (28.4 L/s) and a 10 percent safety factor, the flow of $450 + 0.10(450) = 495$ gal/min (31.2 L/s) would be rounded to 500 gal/min (31.6 L/s) *before* the pump was selected. A pump of 500-gal/min (31.6-L/s), or larger, capacity would be selected.

3. *Compute the total head on the pump*

Use the steps given in the previous calculation procedure to compute the total head on the pump. Express the result in ft (m) of water—this is the most common way of expressing the head on a pump. Be certain to use the exact specific gravity of the liquid handled when expressing the head in ft (m) of water. A specific gravity less than 1.00 *reduces* the total head when expressed in ft (m) of water; whereas a specific gravity greater than 1.00

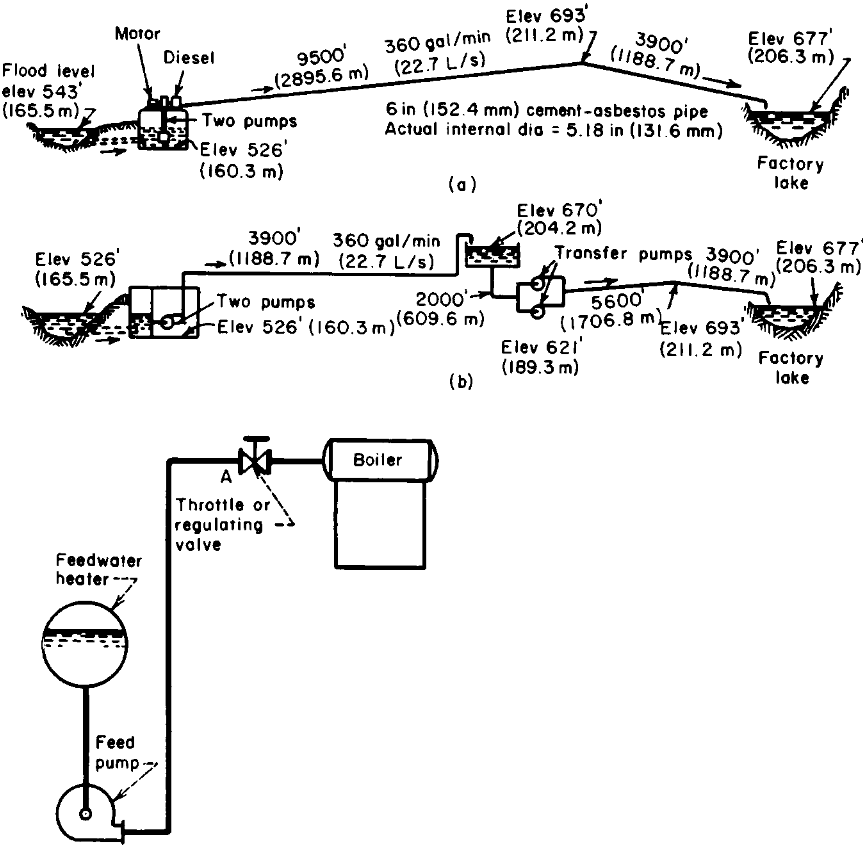


FIGURE 26. (*a*) Single-line diagrams for an industrial pipeline; (*b*) single-line diagram of a boiler-feed system. (*Worthington Corporation.*)

increases the total head when expressed in ft (m) of water. Note that variations in the suction and discharge conditions can affect the total head on the pump.

4. *Analyze the liquid conditions*

Obtain complete data on the liquid pumped. These data should include the name and chemical formula of the liquid, maximum and minimum pumping temperature, corresponding vapor pressure at these temperatures, specific gravity, viscosity at the pumping temperature, pH, flash point, ignition temperature, unusual characteristics (such as tendency to foam, curd, crystallize, become gelatinous or tacky), solids content, type of solids and their size, and variation in the chemical analysis of the liquid.

Enter the liquid conditions on a pump selection form like that in Fig. 27. Such forms are available from many pump manufacturers or can be prepared to meet special job conditions.

Summary of Essential Data Required in Selection of Centrifugal Pumps

1. Number of Units Required

2. Nature of the Liquid to Be Pumped
   Is the liquid:
   a. Fresh or salt water, acid or alkali, oil, gasoline, slurry, or paper stock?
   b. Cold or hot and if hot, at what temperature? What is the vapor pressure of the liquid at the pumping temperature?
   c. What is its specific gravity?
   d. Is it viscous or nonviscous?
   e. Clear and free from suspended foreign matter or dirty and gritty? If the latter, what is the size and nature of the solids, and are they abrasive? If the liquid is of a pulpy nature, what is the consistency expressed either in percentage or in lb per cu ft of liquid? What is the suspended material?
   f. What is the chemical analysis, pH value, etc.? What are the expected variations of this analysis? If corrosive, what has been the past experience, both with successful materials and with unsatisfactory materials?

3. Capacity
   What is the required capacity as well as the minimum and maximum amount of liquid the pump will ever be called upon to deliver?

4. Suction Conditions
   Is there:
   a. A suction lift?
   b. Or a suction head?
   c. What are the length and diameter of the suction pipe?

5. Discharge Conditions
   a. What is the static head? Is it constant or variable?
   b. What is the friction head?
   c. What is the maximum discharge pressure against which the pump must deliver the liquid?

6. Total Head
   Variations in items 4 and 5 will cause variations in the total head.

7. Is the service continuous or intermittent?

8. Is the pump to be installed in a horizontal or vertical position? If the latter,
   a. In a wet pit?
   b. In a dry pit?

9. What type of power is available to drive the pump and what are the characteristics of this power?

10. What space, weight, or transportation limitations are involved?

11. Location of installation
    a. Geographical location
    b. Elevation above sea level
    c. Indoor or outdoor installation
    d. Range of ambient temperatures

12. Are there any special requirements or marked preferences with respect to the design, construction, or performance of the pump?

FIGURE 27. Typical selection chart for centrifugal pumps. (*Worthington Corporation.*)

5. *Select the class and type of pump*

Three *classes* of pumps are used today—centrifugal, rotary, and reciprocating, Fig. 28. Note that these terms apply only to the mechanics of moving the liquid—not to the service for which the pump was designed. Each class of pump is further subdivided into a number of *types*, Fig. 28.

Use Table 5 as a general guide to the class and type of pump to be used. For example, when a large capacity at moderate pressure is required, Table 5 shows that a centrifugal

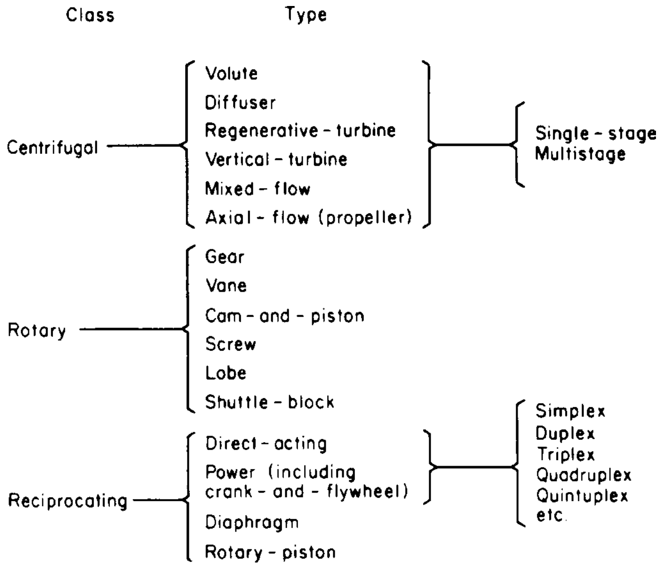


FIGURE 28. Modern pump classes and types.

pump would probably be best. Table 5 also shows the typical characteristics of various classes and types of pumps used in industrial process work.

Consider the liquid properties when choosing the class and type of pump, because exceptionally severe conditions may rule out one or another class of pump at the start. Thus, screw- and gear-type rotary pumps are suitable for handling viscous, nonabrasive liquid, Table 5. When an abrasive liquid must be handled, either another class of pump or another type of rotary pump must be used.

Also consider all the operating factors related to the particular pump. These factors include the type of service (continuous or intermittent), operating-speed preferences, future load expected and its effect on pump head and capacity, maintenance facilities available, possibility of parallel or series hookup, and other conditions peculiar to a given job.

Once the class and type of pump is selected, consult a rating table (Table 6) or rating chart, Fig. 29, to determine whether a suitable pump is available from the manufacturer whose unit will be used. When the hydraulic requirements fall between two standard pump models, it is usual practice to choose the next larger size of pump, unless there is some reason why an exact head and capacity are required for the unit. When one manufacturer does not have the desired unit, refer to the engineering data of other manufacturers. Also keep in mind that some pumps are custom-built for a given job when precise head and capacity requirements must be met.

Other pump data included in manufacturer's engineering information include characteristic curves for various diameter impellers in the same casing, Fig. 30, and variable-speed head-capacity curves for an impeller of given diameter, Fig. 31. Note that the required power input is given in Figs. 29 and 30 and may also be given in Fig. 31. Use of Table 6 is explained in the table.

Performance data for rotary pumps are given in several forms. Figure 32 shows a typical plot of the head and capacity ranges of different types of rotary pumps. Reciprocating-pump capacity data are often tabulated, as in Table 7.

TABLE 5. Characteristics of Modern Pumps

	Centrifugal		Rotary	Reciprocating		
	Volute and diffuser	Axial flow	Screw and gear	Direct acting steam	Double acting power	Triplex
Discharge flow	Steady	Steady	Steady	Pulsating	Pulsating	Pulsating
Usual maximum suction lift, ft (m)	15 (4.6)	15 (4.6)	22 (6.7)	22 (6.7)	22 (6.7)	22 (6.7)
Liquids handled	Clean, clear; dirty, abrasive; liquids with high solids content		Viscous; non-abrasive	Clean and clear		
Discharge pressure range	Low to high		Medium	Low to highest produced		
Usual capacity range	Small to largest available		Small to medium	Relatively small		
How increased head affects:						
Capacity	Decrease		None	Decrease	None	None
Power input	Depends on specific speed		Increase	Increase	Increase	Increase
How decreased head affects:						
Capacity	Increase		None	Small increase	None	None
Power input	Depends on specific speed		Decrease	Decrease	Decrease	Decrease

TABLE 6. Typical Centrifugal-Pump Rating Table

Size		Total head			
gal/min	L/s	20 ft, 4/min—hp	6.1 m, r/min—kW	25 ft, r/min—hp	7.6 m, r/min—kW
3 CL:					
200	12.6	910—1.3	910—0.97	1010—1.6	1010—1.19
300	18.9	1000—1.9	1000—1.41	1100—2.4	1100—1.79
400	25.2	1200—3.1	1200—2.31	1230—3.7	1230—2.76
500	31.5	—	—	—	—
4 C:					
400	25.2	940—2.4	940—1.79	1040—3	1040—2.24
600	37.9	1080—4	1080—2.98	1170—4.6	1170—3.43
800	50.5	—	—	—	—

Example: 1080—4 indicates pump speed is 1080 r/min; actual input required to operate the pump is 4 hp (2.98 kW).

Source: Condensed from data of Goulds Pumps, Inc.; SI values added by handbook editor.

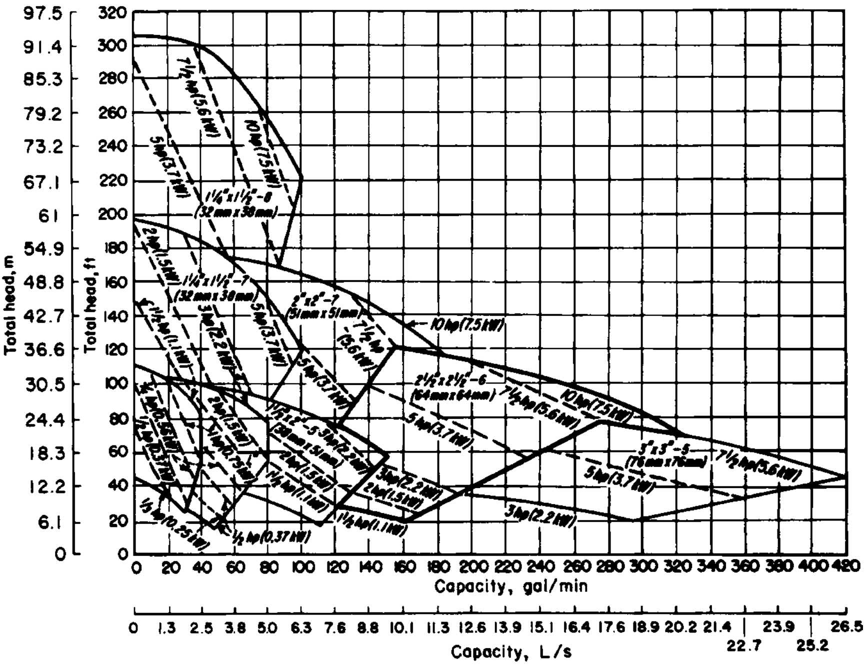


FIGURE 29. Composite rating chart for a typical centrifugal pump. (*Goulds Pumps, Inc.*)

6. *Evaluate the pump chosen for the installation*

Check the specific speed of a centrifugal pump, using the method given in an earlier calculation procedure. Once the specific speed is known, the impeller type and approximate operating efficiency can be found from Fig. 23.

Check the piping system, using the method of an earlier calculation procedure, to see whether the available net positive suction head equals, or is greater than, the required net positive suction head of the pump.

Determine whether a vertical or horizontal pump is more desirable. From the standpoint of floor space occupied, required NPSH, priming, and flexibility in changing the pump use, vertical pumps may be preferable to horizontal designs in some installations. But where headroom, corrosion, abrasion, and ease of maintenance are important factors, horizontal pumps may be preferable.

As a general guide, single-suction centrifugal pumps handle up to 50 gal/min (3.2 L/s) at total heads up to 50 ft (15.2 m); either single- or double-suction pumps are used for the flow rates to 1000 gal/min (63.1 L/s) and total heads to 300 ft (91.4 m); beyond these capacities and heads, double-suction or multistage pumps are generally used.

Mechanical seals are becoming more popular for all types of centrifugal pumps in a variety of services. Although they are more costly than packing, the mechanical seal reduces pump maintenance costs.

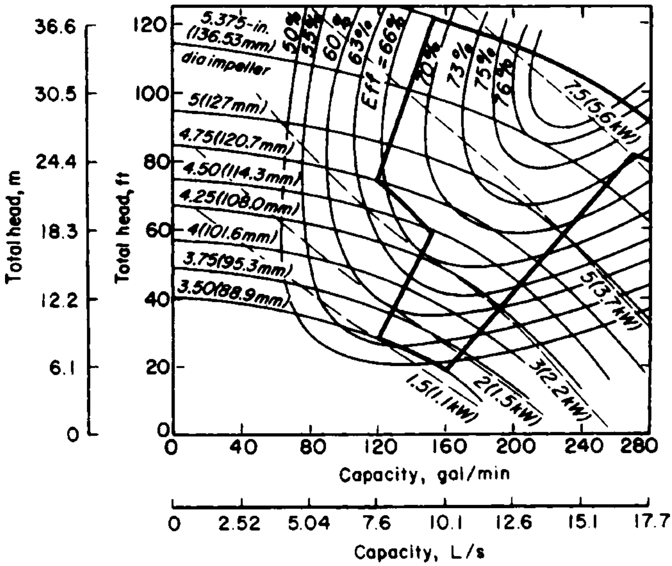


FIGURE 30. Pump characteristics when impeller diameter is varied within the same casing.

Related Calculations. Use the procedure given here to select any class of pump—centrifugal, rotary, or reciprocating—for any type of service—power plant, atomic energy, petroleum processing, chemical manufacture, paper mills, textile mills, rubber factories, food processing, water supply, sewage and sump service, air conditioning and heating, irrigation and flood control, mining and construction, marine services, industrial hydraulics, iron and steel manufacture.

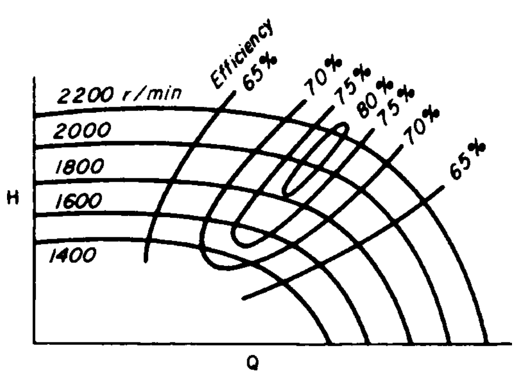


FIGURE 31. Variable-speed head-capacity curves for a centrifugal pump.

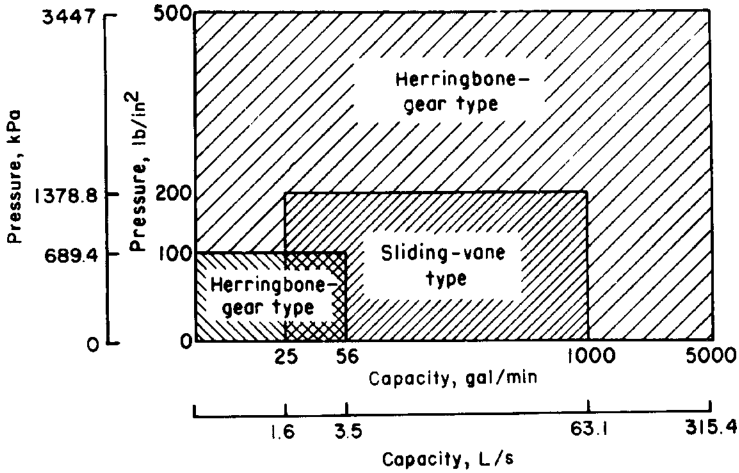


FIGURE 32. Capacity ranges of some rotary pumps. (*Worthington Corporation.*)

TABLE 7. Capacities of Typical Horizontal Duplex Plunger Pumps

Size		Cold-water pressure service			
				Piston speed	
in.	cm	gal/min	L/s	ft/min	m/min
6 × 3½ × 6	15.2 × 8.9 × 15.2	60	3.8	60	18.3
7½ × 4½ × 10	19.1 × 11.4 × 25.4	124	7.8	75	22.9
9 × 5 × 10	22.9 × 12.7 × 25.4	153	9.7	75	22.9
10 × 6 × 12	25.4 × 15.2 × 30.5	235	14.8	80	24.4
12 × 7 × 12	30.5 × 17.8 × 30.5	320	20.2	80	24.4

Size		Boiler-feed service					
				Boiler		Piston speed	
in.	cm	gal/min	L/s	hp	kW	ft/min	m/min
6 × 3½ × 6	15.2 × 8.9 × 15.2	36	2.3	475	354.4	36	10.9
7½ × 4½ × 10	19.1 × 11.4 × 25.4	74	4.7	975	727.4	45	13.7
9 × 5 × 10	22.9 × 12.7 × 25.4	92	5.8	1210	902.7	45	13.7
10 × 6 × 12	25.4 × 15.2 × 30.5	141	8.9	1860	1387.6	48	14.6
12 × 7 × 12	30.5 × 17.8 × 30.5	192	12.1	2530	1887.4	48	14.6

Source: Courtesy of Worthington Corporation.

ANALYSIS OF PUMP AND SYSTEM CHARACTERISTIC CURVES

Analyze a set of pump and system characteristic curves for the following conditions: friction losses without static head; friction losses with static head; pump without lift; system with little friction; much static head; system with gravity head; system with different pipe sizes; system with two discharge heads; system with diverted flow; and effect of pump wear on characteristic curve.

Calculation Procedure:

1. *Plot the system-friction curve*

Without static head, the system-friction curve passes through the origin (0,0), Fig. 33, because when no head is developed by the pump, flow through the piping is zero. For most piping systems, the friction-head loss varies as the square of the liquid flow rate in the system. Hence, a system-friction curve, also called a friction-head curve, is parabolic—the friction head increases as the flow rate or capacity of the system increases. Draw the curve as shown in Fig. 33.

2. *Plot the piping system and system-head curve*

Figure 34*a* shows a typical piping system with a pump operating against a static discharge head. Indicate the total static head, Fig. 34*b*, by a dashed line—in this installation H_{ts} = 110 ft. Since static head is a physical dimension, it does not vary with flow rate and is a constant for all flow rates. Draw the dashed line parallel to the abscissa, Fig. 34*b*.

From the point of no flow—zero capacity—plot the friction-head loss at various flow rates—100, 200, 300 gal/min (6.3, 12.6, 18.9 L/s), etc. Determine the friction-head loss by computing it as shown in an earlier calculation procedure. Draw a curve through the points obtained. This is called the *system-head curve.*

Plot the pump head-capacity (*H-Q*) curve of the pump on Fig. 34*b*. The *H-Q* curve can be obtained from the pump manufacturer or from a tabulation of *H* and *Q* values for the pump being considered. The point of intersection *A* between the *H-Q* and system-head curves is the operating point of the pump.

Changing the resistance of a given piping system by partially closing a valve or making some other change in the friction alters the position of the system-head curve and pump operating point. Compute the frictional resistance as before, and plot the artificial system-head curve as shown. Where this curve intersects the *H-Q* curve is the new operating point of the pump. System-head curves are valuable for analyzing the suitability of a given pump for a particular application.

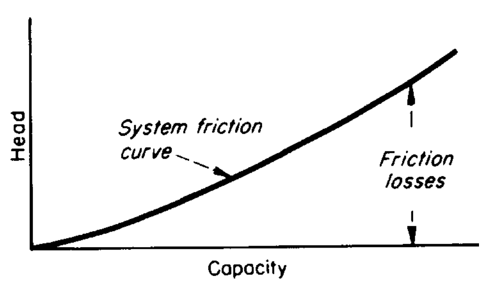


FIGURE 33. Typical system-friction curve.

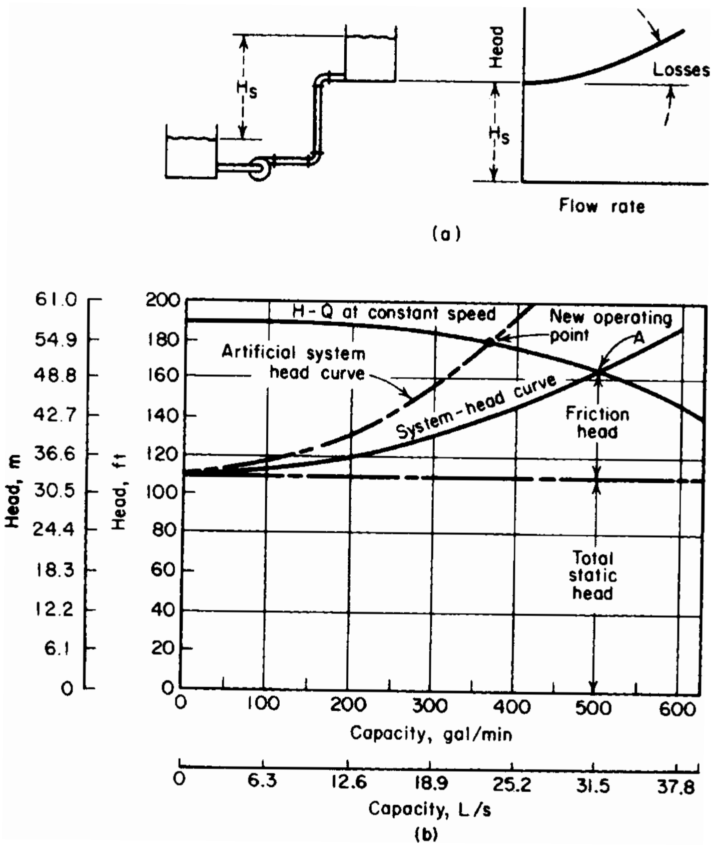


FIGURE 34. (*a*) Significant friction loss and lift; (*b*) system-head curve superimposed on pump head-capacity curve. (*Peerless Pumps.*)

3. *Plot the no-lift system-head curve and compute the losses*

With no static head or lift, the system-head curve passes through the origin (0,0), Fig. 35. For a flow of 900 gal/min (56.8 L/s) in this system, compute the friction loss as follows, using the Hydraulic Institute *Pipe Friction Manual* tables or the method of earlier calculation procedures:

	ft	m
Entrance loss from tank into 10-in. (254-mm) suction pipe, $0.5v^2/2g$	0.10	0.03
Friction loss in 2 ft (0.61 m) of suction pipe	0.02	0.01
Loss in 10-in. (254-mm) 90° elbow at pump	0.20	0.06
Friction loss in 3000 ft (914.4 m) of 8-in. (203.2-mm) discharge pipe	74.50	22.71
Loss in fully open 8-in. (203.2-mm) gate valve	0.12	0.04
Exit loss from 8-in. (203.2-mm) pipe into tank, $v^2/2g$	0.52	0.16
Total friction loss	75.46	23.01

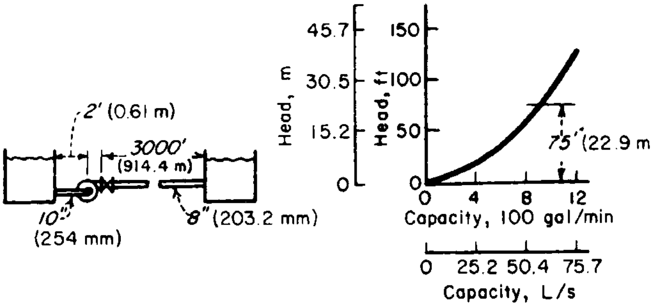


FIGURE 35. No lift; all friction head. (*Peerless Pumps.*)

Compute the friction loss at other flow rates in a similar manner, and plot the system-head curve, Fig. 35. Note that if all losses in this system except the friction in the discharge pipe were ignored, the total head would not change appreciably. However, for the purposes of accuracy, all losses should always be computed.

4. *Plot the low-friction, high-head system-head curve*

The system-head curve for the vertical pump installation in Fig. 36 starts at the total static head, 15 ft (4.6 m), and zero flow. Compute the friction head for 15,000 gal/min as follows:

	ft	m
Friction in 20 ft (6.1 m) of 24-in. (609.6-mm) pipe	0.40	0.12
Exit loss from 24-in. (609.6-mm) pipe into tank, $v^2/2g$	1.60	0.49
Total friction loss	2.00	0.61

Hence, almost 90 percent of the total head of 15 + 2 = 17 ft (5.2 m) at 15,000-gal/min (946.4-L/s) flow is static head. But neglect of the pipe friction and exit losses could cause appreciable error during selection of a pump for the job.

5. *Plot the gravity-head system-head curve*

In a system with gravity head (also called negative lift), fluid flow will continue until the system friction loss equals the available gravity head. In Fig. 37 the available gravity head

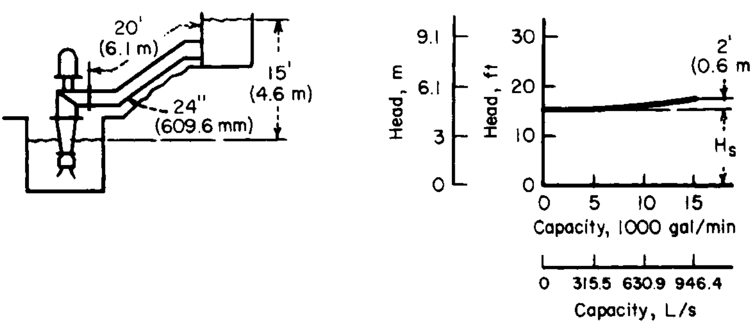


FIGURE 36. Mostly lift; little friction head. (*Peerless Pumps.*)

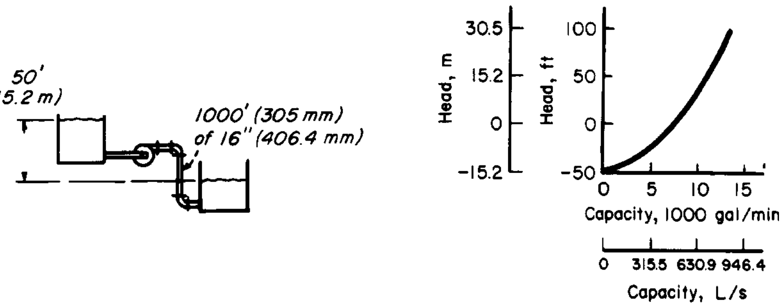


FIGURE 37. Negative lift (gravity head). (*Peerless Pumps.*)

is 50 ft (15.2 m). Flows up to 7200 gal/min (454.3 L/s) are obtained by gravity head alone. To obtain larger flow rates, a pump is needed to overcome the friction in the piping between the tanks. Compute the friction loss for several flow rates as follows:

	ft	m
At 5000 gal/min (315.5 L/s) friction loss in 1000 ft (305 m) of 16-in. (406.4-mm) pipe	25	7.6
At 7200 gal/min (454.3 L/s), friction loss = available gravity head	50	15.2
At 13,000 gal/min (820.2 L/s), friction loss	150	45.7

Using these three flow rates, plot the system-head curve, Fig. 37.

6. *Plot the system-head curves for different pipe sizes*

When different diameter pipes are used, the friction loss vs. flow rate is plotted independently for the two pipe sizes. At a given flow rate, the total friction loss for the system is the sum of the loss for the two pipes. Thus, the combined system-head curve represents the sum of the static head and the friction losses for all portions of the pipe.

Figure 38 shows a system with two different pipe sizes. Compute the friction losses as follows:

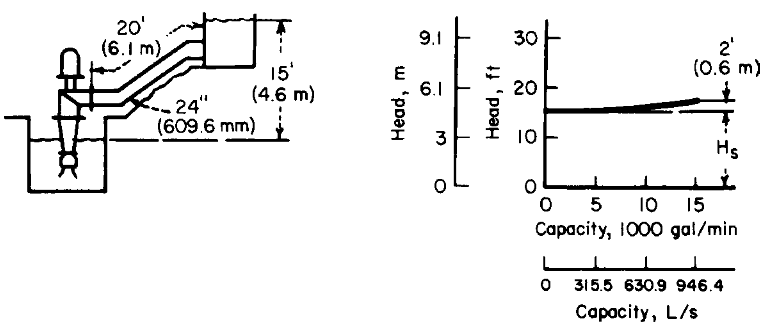


FIGURE 38. System with two different pipe sizes. (*Peerless Pumps.*)

	ft	m
At 150 gal/min (9.5 L/s), friction loss in 200 ft (60.9 m) of 4-in. (102-mm) pipe	5	1.52
At 150 gal/min (9.5 L/s), friction loss in 200 ft (60.9 m) of 3-in. (76.2-mm) pipe	19	5.79
Total static head for 3- (76.2-) and 4-in. (102-mm) pipes	10	3.05
Total head at 150-gal/min (9.5-L/s) flow	34	10.36

Compute the total head at other flow rates, and then plot the system-head curve as shown in Fig. 38.

7. *Plot the system-head curve for two discharge heads*

Figure 39 shows a typical pumping system having two different discharge heads. Plot separate system-head curves when the discharge heads are different. Add the flow rates for the two pipes at the same head to find points on the combined system-head curve, Fig. 39. Thus,

	ft	m
At 550 gal/min (34.7 L/s), friction loss in 1000 ft (305 m) of 8-in. (203.2-mm) pipe	10	3.05
At 1150 gal/min (72.6 L/s) friction	38	11.6
At 1150 gal/min (72.6 L/s), friction + lift in pipe 1	88	26.8
At 550 gal/min (34.7 L/s), friction + lift in pipe 2	88	26.8

The flow rate for the combined system at a head of 88 ft (26.8 m) is 1150 + 550 = 1700 gal/min (107.3 L/s). To produce a flow of 1700 gal/min (107.3 L/s) through this system, a pump capable of developing an 88-ft (26.8-m) head is required.

8. *Plot the system-head curve for diverted flow*

To analyze a system with diverted flow, assume that a constant quantity of liquid is tapped off at the intermediate point. Plot the friction loss vs. flow rate in the normal manner for pipe 1, Fig. 40. Move the curve for pipe 3 to the right at zero head by an amount equal to Q_2, since this represents the quantity passing through pipes 1 and 2 but

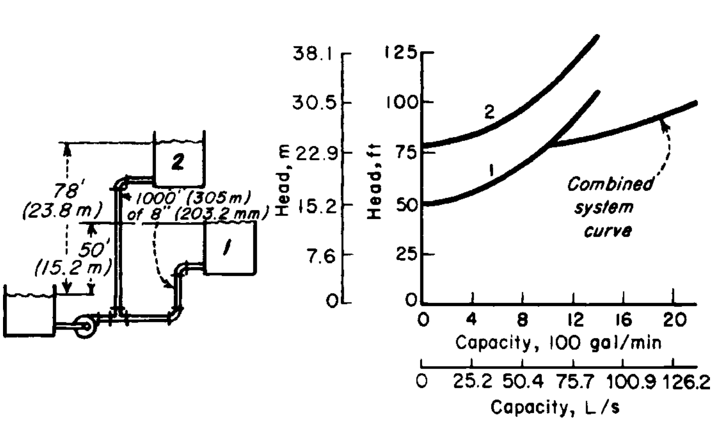


FIGURE 39. System with two different discharge heads. (*Peerless Pumps.*)

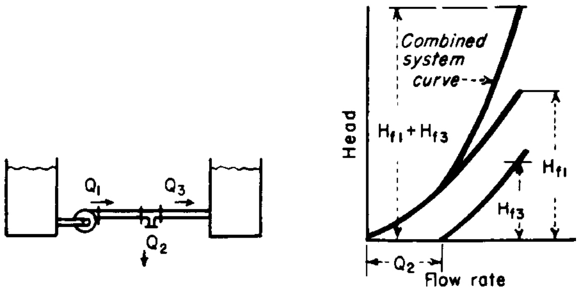


FIGURE 40. Part of the fluid flow is diverted from the main pipe. (*Peerless Pumps.*)

not through pipe 3. Plot the combined system-head curve by adding, at a given flow rate, the head losses for pipes 1 and 3. With Q = 300 gal/min (18.9 L/s), pipe 1 = 500 ft (152.4 m) of 10-in. (254-mm) pipe, and pipe 3 = 50 ft (15.2 m) of 6-in. (152.4-mm) pipe.

	ft	m
At 1500 gal/min (94.6 L/s) through pipe 1, friction loss	11	3.35
Friction loss for pipe 3 (1500 − 300 = 1200 gal/min) (75.7 L/s)	8	2.44
Total friction loss at 1500-gal/min (94.6-L/s) delivery	19	5.79

9. *Plot the effect of pump wear*

When a pump wears, there is a loss in capacity and efficiency. The amount of loss depends, however, on the shape of the system-head curve. For a centrifugal pump, Fig. 41, the capacity loss is greater for a given amount of wear if the system-head curve is flat, as compared with a steep system-head curve.

Determine the capacity loss for a worn pump by plotting its *H-Q* curve. Find this curve by testing the pump at different capacities and plotting the corresponding head. On the same chart, plot the *H-Q* curve for a new pump of the same size, Fig. 41. Plot the system-head curve, and determine the capacity loss as shown in Fig. 41.

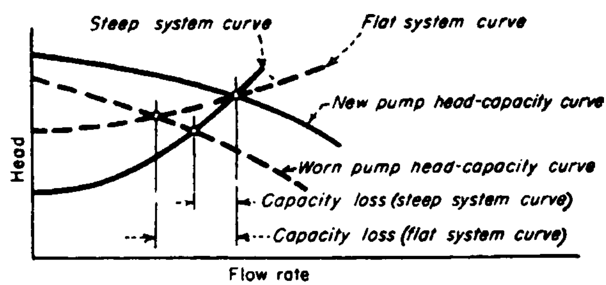


FIGURE 41. Effect of pump wear on pump capacity. (*Peerless Pumps.*)

Related Calculations. Use the techniques given here for any type of pump—centrifugal, reciprocating, or rotary—handling any type of liquid—oil, water, chemicals, etc. The methods given here are the work of Melvin Mann, as reported in *Chemical Engineering,* and Peerless Pump Division of FMC Corp.

NET POSITIVE SUCTION HEAD FOR HOT-LIQUID PUMPS

What is the maximum capacity of a double-suction pump operating at 1750 r/min if it handles 100°F (37.8°C) water from a hot well having an absolute pressure of 2.0 in. (50.8 mm) Hg if the pump centerline is 10 ft (30.5 m) below the hot-well liquid level and the friction-head loss in the suction piping and fitting is 5 ft (1.52 m) of water?

Calculation Procedure:

1. *Compute the net positive suction head on the pump*

The net positive suction head h_n on a pump when the liquid supply is *above* the pump inlet = pressure on liquid surface + static suction head − friction-head loss in suction piping and pump inlet − vapor pressure of the liquid, all expressed in ft absolute of liquid handled. When the liquid supply is *below* the pump centerline—i.e., there is a static suction lift—the vertical distance of the lift is *subtracted* from the pressure on the liquid surface instead of added as in the above relation.

The density of 100°F (37.8°C) water is 62.0 lb/ft^3 (992.6 kg/m^3), computed as shown in earlier calculation procedures in this handbook. The pressure on the liquid surface, in absolute ft of liquid = (2.0 in Hg)(1.133)(62.4/62.0) = 2.24 ft (0.68 m). In this calculation, 1.133 = ft of 39.2°F (4°C) water = 1 in Hg; 62.4 = lb/ft^3 (999.0 kg/m^3) of 39.2°F (4°C) water. The temperature of 39.2°F (4°C) is used because at this temperature water has its maximum density. Thus, to convert in Hg to ft absolute of water, find the product of (in Hg)(1.133)(water density at 39.2°F)/(water density at operating temperature). Express both density values in the same unit, usually lb/ft^3.

The static suction head is a physical dimension that is measured in ft (m) of liquid at the operating temperature. In this installation, h_{sh} = 10 ft (3 m) absolute.

The friction-head loss is 5 ft (1.52 m) of water. When it is computed by using the methods of earlier calculation procedures, this head loss is in ft (m) of water at maximum density. To convert to ft absolute, multiply by the ratio of water densities at 39.2°F (4°C) and the operating temperature, or (5)(62.4/62.0) = 5.03 ft (1.53 m).

The vapor pressure of water at 100°F (37.8°C) is 0.949 lb/sq.in. (abs) (6.5 kPa) from the steam tables. Convert any vapor pressure to ft absolute by finding the result of [vapor pressure, lb/sq.in. (abs)] (144 sq.in./sq.ft.)/liquid density at operating temperature, or (0.949)(144)/62.0 = 2.204 ft (0.67 m) absolute.

With all the heads known, the net positive suction head is $h_n = 2.24 + 10 - 5.03 - 2.204 = 5.01$ ft (1.53 m) absolute.

2. *Determine the capacity of the pump*

Use the Hydraulic Institute curve, Fig. 42, to determine the maximum capacity of the pump. Enter at the left of Fig. 42 at a net positive suction head of 5.01 ft (1.53 m), and project horizontally to the right until the 3500-r/min curve is intersected. At the top, read the capacity as 278 gal/min (17.5 L/s).

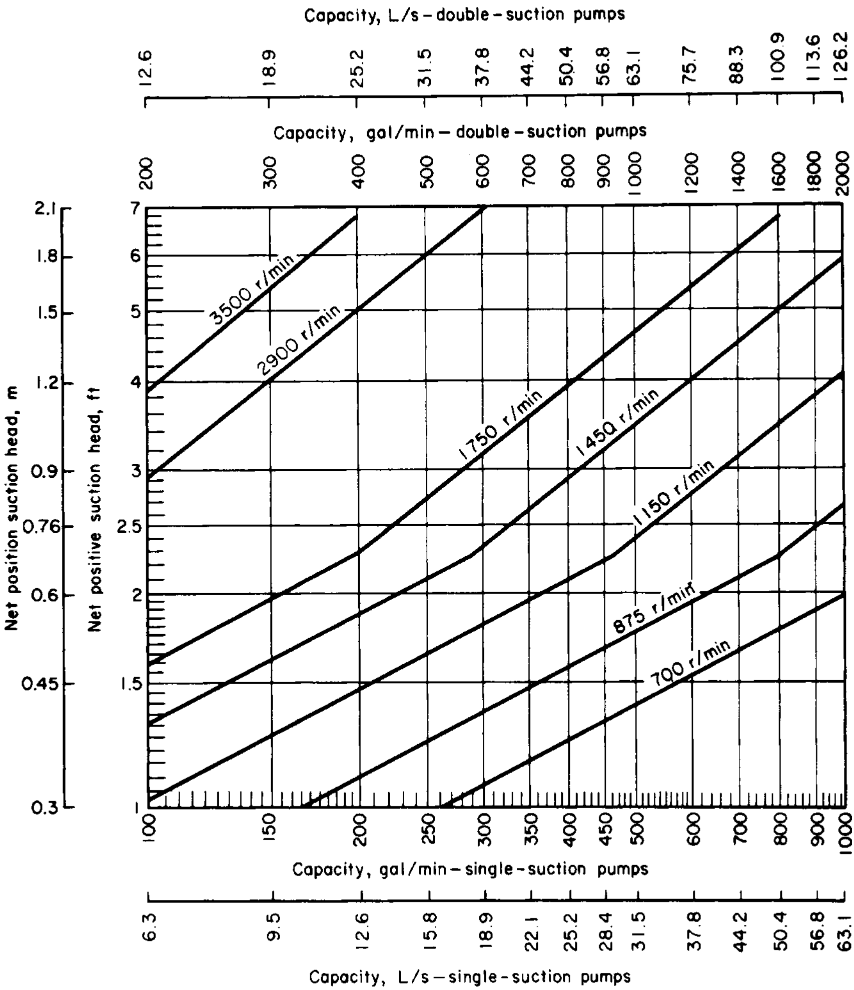


FIGURE 42. Capacity and speed limitations of pumps with the shaft through the impeller eye. (*Hydraulic Institute.*)

Related Calculations. Use this procedure for any pump handling water at an elevated temperature. Consult the *Standards of the Hydraulic Institute* for capacity curves of pumps having different types of construction. In general, pump manufacturers who are members of the Hydraulic Institute rate their pumps in accordance with the *Standards*, and a pump chosen from a catalog capacity table or curve will deliver the stated capacity. A similar procedure is used for computing the capacity of pumps handling volatile petroleum liquids. When you use this procedure, be certain to refer to the latest edition of the *Standards*.

PART 3

CENTRIFUGAL PUMPS AND HYDRO POWER

MINIMUM SAFE FLOW FOR A CENTRIFUGAL PUMP

A centrifugal pump handles 220°F (104.4°C) water and has a shutoff head (with closed discharge valve) of 3200 ft (975.4 m). At shutoff, the pump efficiency is 17 percent and the input brake horsepower is 210 (156.7 kW). What is the minimum safe flow through this pump to prevent overheating at shutoff? Determine the minimum safe flow if the NPSH is 18.8 ft (5.7 m) of water and the liquid specific gravity is 0.995. If the pump contains 500 lb (225 kg) of water, determine the rate of the temperature rise at shutoff.

Calculation Procedure:

1. ***Compute the temperature rise in the pump***

With the discharge valve closed, the power input to the pump is converted to heat in the casing and causes the liquid temperature to rise. The temperature rise $t = (1 - e) \times H_s/778e$, where t = temperature rise during shutoff, °F; e = pump efficiency, expressed as a decimal; H_s = shutoff head, ft. For this pump, $t = (1 - 0.17)(3200)/[7780(0.17)] = 20.4$°F (36.7°C).

2. ***Compute the minimum safe liquid flow***

For general-service pumps, the minimum safe flow M gal/min = 6.0(bhp input at shutoff)/t. Or, $M = 6.0(210)/20.4 = 62.7$ gal/min (3.96 L/s). This equation includes a 20 percent safety factor.

Centrifugal pumps usually have a maximum allowable temperature rise of 15°F (27°C). The minimum allowable flow through the pump to prevent the water temperature from rising more than 15°F (27°C) is 30 gal/min (1.89 L/s) for each 110-bhp (74.6-kW) input at shutoff.

3. ***Compute the temperature rise for the operating NPSH***

An NPSH of 18.8 ft (5.73 m) is equivalent to a pressure of $18.8(0.433)(0.995) = 7.78$ lb/sq.in. (abs) (53.6 kPa) at 220°F (104.4°C), where the factor 0.433 converts ft of water to lb/sq.in. At 220°F (104.4°C), the vapor pressure of the water is 17.19 lb/sq.in. (abs) (118.5 kPa), from the steam tables. Thus, the total vapor pressure the water can develop before flashing occurs = NPSH pressure + vapor pressure at operating temperature = $7.78 + 17.19 = 24.97$ lb/sq.in. (abs) (172.1 kPa). Enter the steam tables at this pressure, and read the corresponding temperature as 240°F (115.6°C). The allowable temperature rise of the water is then $240 - 220 = 20$°F (36.0°C). Using the safe-flow relation of step 2, we find the minimum safe flow is 62.9 gal/min (3.97 L/s).

4. ***Compute the rate of temperature rise***

In any centrifugal pump, the rate of temperature rise t_r °F/min = 42.4(bhp input at shutoff)/wc, where w = weight of liquid in the pump, lb; c = specific heat of the liquid in the pump, Btu/(lb·°F). For this pump containing 500 lb (225 kg) of water with a specific heat, $c = 1.0$, $t_r = 42.4(210)/[500(1.0)] = 17.8$°F/min (32°C/min). This is a very rapid temperature rise and could lead to overheating in a few minutes.

Related Calculations. Use this procedure for any centrifugal pump handling any liquid in any service—power, process, marine, industrial, or commercial. Pump manufacturers can supply a temperature-rise curve for a given model pump if it is requested. This curve is superimposed on the pump characteristic curve and shows the temperature rise accompanying a specific flow through the pump.

SELECTING A CENTRIFUGAL PUMP TO HANDLE A VISCOUS LIQUID

Select a centrifugal pump to deliver 750 gal/min (47.3 L/s) of 1000-SSU oil at a total head of 100 ft (30.5 m). The oil has a specific gravity of 0.90 at the pumping temperature. Show how to plot the characteristic curves when the pump is handling the viscous liquid.

Calculation Procedure:

1. *Determine the required correction factors*

A centrifugal pump handling a viscous liquid usually must develop a greater capacity and head, and it requires a larger power input than the same pump handling water. With the water performance of the pump known—from either the pump characteristic curves or a tabulation of pump performance parameters—Fig. 43, prepared by the Hydraulic Institute, can be used to find suitable correction factors. Use this chart only within its scale limits; do not extrapolate. Do not use the chart for mixed-flow or axial-flow pumps or for pumps of special design. Use the chart only for pumps handling uniform liquids; slurries, gels, paper stock, etc., may cause incorrect results. In using the chart, the available net positive suction head is assumed adequate for the pump.

To use Fig. 43, enter at the bottom at the required capacity, 750 gal/min (47.3 L/s), and project vertically to intersect the 100-ft (30.5-m) head curve, the required head. From here project horizontally to the 1000-SSU viscosity curve, and then vertically upward to the correction-factor curves. Read $C_E = 0.635$; $C_Q = 0.95$; $C_H = 0.92$ for $1.0Q_{NW}$. The subscripts E, Q, and H refer to correction factors for efficiency, capacity, and head, respectively; and NW refers to the water capacity at a particular efficiency. At maximum efficiency, the water capacity is given as $1.0Q_{NW}$; other efficiencies, expressed by numbers equal to or less than unity, give different capacities.

2. *Compute the water characteristics required*

The water capacity required for the pump $Q_w = Q_v/C_Q$ where Q_v = viscous capacity, gal/min. For this pump, $Q_w = 750/0.95 = 790$ gal/min (49.8 L/s). Likewise, water head $H_w = H_v/C_H$, where H_v = viscous head. Or, $H_w = 100/0.92 = 108.8$ (33.2 m), say 109 ft (33.2 m) of water.

Choose a pump to deliver 790 gal/min (49.8 L/s) of water at 109-ft (33.2-m) head of water, and the required viscous head and capacity will be obtained. Pick the pump so that it is operating at or near its maximum efficiency on water. If the water efficiency $E_w = 81$ percent at 790 gal/min (49.8 L/s) for this pump, the efficiency when handling the viscous liquid $E_v = E_wC_E$. Or, $E_v = 0.81(0.635) = 0.515$, or 51.5 percent.

The power input to the pump when handling viscous liquids is given by $P_v = Q_vH_vs/3960E_v$, where s = specific gravity of the viscous liquid. For this pump, $P_v = (750) \times (100)(0.90)/[3960(0.515)] = 33.1$ hp (24.7 kW).

3. *Plot the characteristic curves for viscous-liquid pumping*

Follow these eight steps to plot the complete characteristic curves of a centrifugal pump handling a viscous liquid when the water characteristics are known: (*a*) Secure a complete

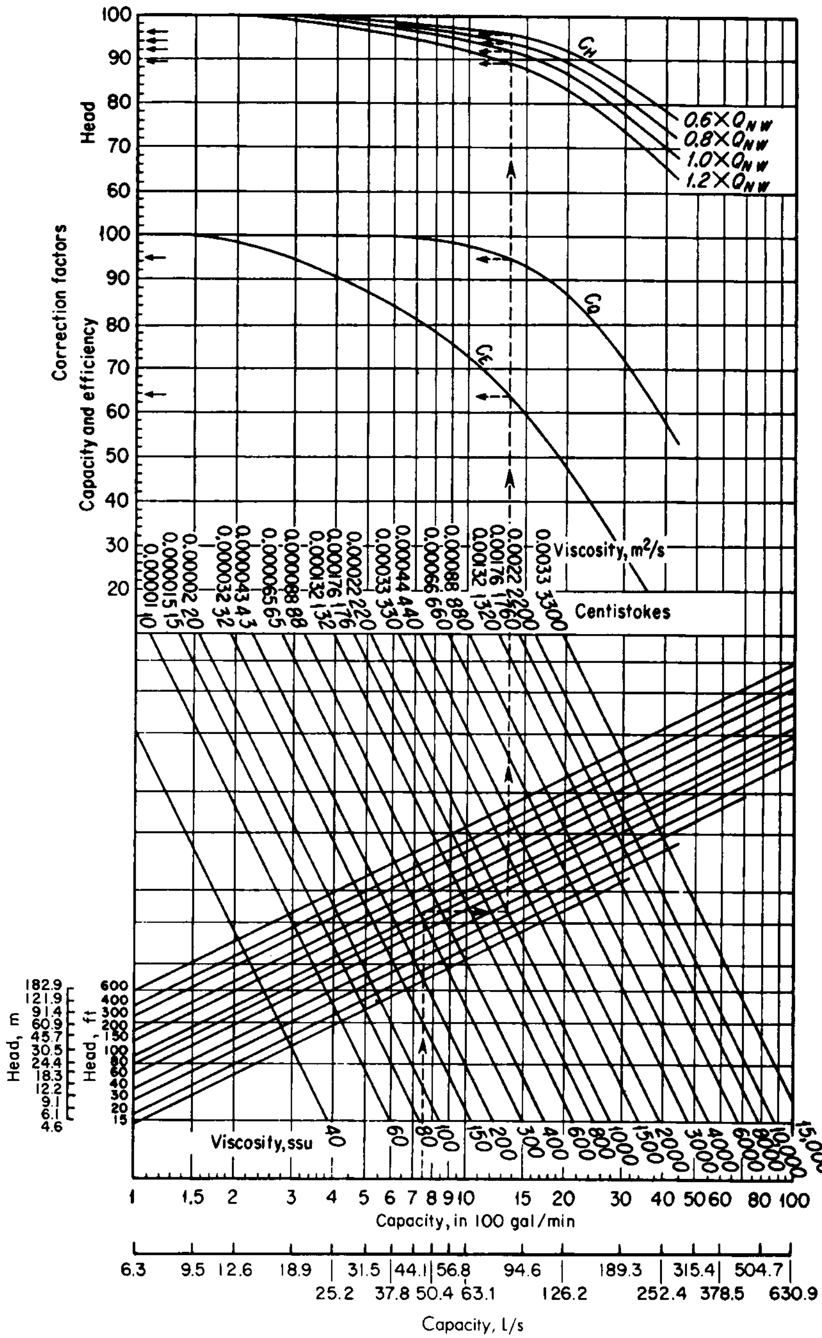


FIGURE 43. Correction factors for viscous liquids handled by centrifugal pumps. (*Hydraulic Institute.*)

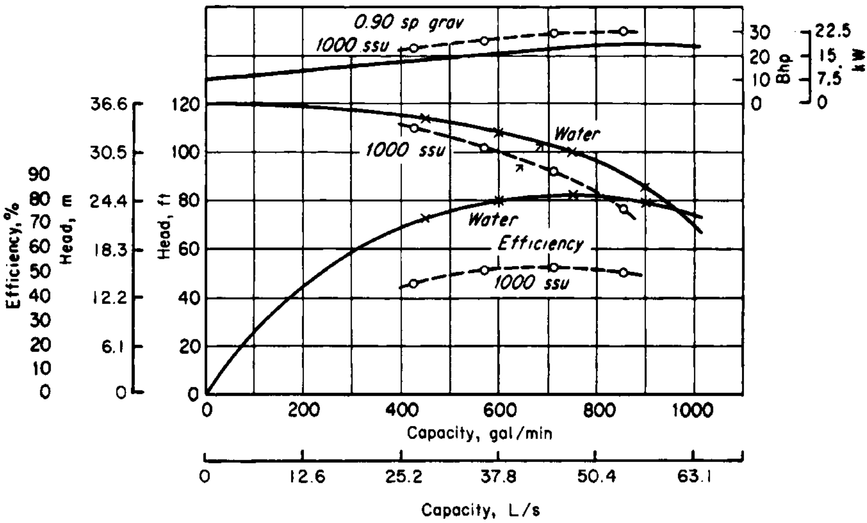


FIGURE 44. Characteristic curves for water (solid line) and oil (dashed line). (*Hydraulic Institute.*)

set of characteristic curves (H, Q, P, E) for the pump to be used. (*b*) Locate the point of maximum efficiency for the pump when handling water. (*c*) Read the pump capacity, Q gal/min, at this point. (*d*) Compute the values of $0.6Q$, $0.8Q$, and $1.2Q$ at the maximum efficiency. (*e*) Using Fig. 43, determine the correction factors at the capacities in steps *c* and *d*. Where a multistage pump is being considered, use the head per stage (= total pump head, ft/number of stages), when entering Fig. 43. (*f*). Correct the head, capacity, and efficiency for each of the flow rates in *c* and *d*, using the correction factors from Fig. 43. (*g*) Plot the corrected head and efficiency against the corrected capacity, as in Fig. 43. (*h*) Compute the power input at each flow rate and plot. Draw smooth curves through the points obtained, Fig. 44.

Related Calculations. Use the method given here for any uniform viscous liquid—oil, gasoline, kerosene, mercury, etc.—handled by a centrifugal pump. Be careful to use Fig. 1 only within its scale limits; *do not extrapolate.* The method presented here is that developed by the Hydraulic Institute. For new developments in the method, be certain to consult the latest edition of the Hydraulic Institute *Standards*.

PUMP SHAFT DEFLECTION AND CRITICAL SPEED

What are the shaft deflection and approximate first critical speed of a centrifugal pump if the total combined weight of the pump impellers is 23 lb (10.4 kg) and the pump manufacturer supplies the engineering data in Fig. 45?

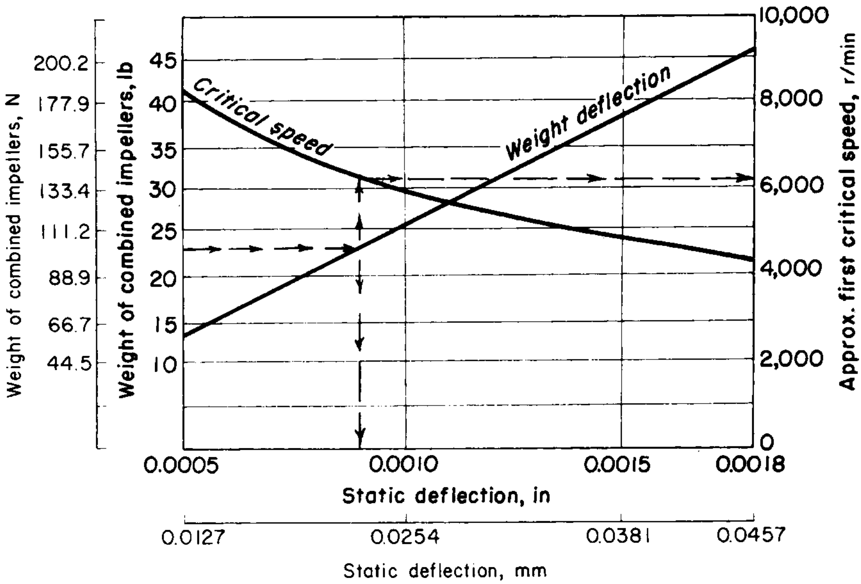


FIGURE 45. Pump shaft deflection and critical speed. (*Goulds Pumps, Inc.*)

Calculation Procedure:

1. *Determine the deflection of the pump shaft*

Use Fig. 45 to determine the shaft deflection. Note that this chart is valid for only one pump or series of pumps and must be obtained from the pump builder. Such a chart is difficult to prepare from test data without extensive test facilities.

Enter Fig. 45 at the left at the total combined weight of the impellers, 23 lb (10.4 kg), and project horizontally to the right until the weight-deflection curve is intersected. From the intersection, project vertically downward to read the shaft deflection as 0.009 in. (0.23 mm) at full speed.

2. *Determine the critical speed of the pump*

From the intersection of the weight-deflection curve in Fig. 45 project vertically upward to the critical-speed curve. Project horizontally right from this intersection and read the first critical speed as 6200 r/min.

Related Calculations. Use this procedure for any class of pump—centrifugal, rotary, or reciprocating—for which the shaft-deflection and critical-speed curves are available. These pumps can be used for any purpose—process, power, marine, industrial, or commercial.

EFFECT OF LIQUID VISCOSITY ON REGENERATIVE-PUMP PERFORMANCE

A regenerative (turbine) pump has the water head-capacity and power-input characteristics shown in Fig. 46. Determine the head-capacity and power-input characteristics for

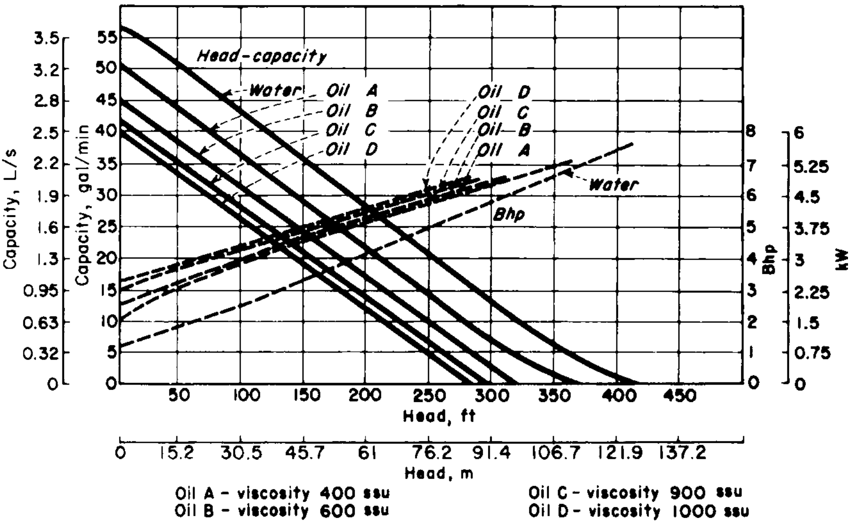


FIGURE 46. Regenerative pump performance when handling water and oil. (*Aurora Pump Division, The New York Air Brake Company.*)

four different viscosity oils to be handled by the pump—400, 600, 900, and 1000 SSU. What effect does increased viscosity have on the performance of the pump?

Calculation Procedure:

1. *Plot the water characteristics of the pump*

Obtain a tabulation or plot of the water characteristics of the pump from the manufacturer or from their engineering data. With a tabulation of the characteristics, enter the various capacity and power points given, and draw a smooth curve through them, Fig. 46.

2. *Plot the viscous-liquid characteristics of the pump*

The viscous-liquid characteristics of regenerative-type pumps are obtained by test of the actual unit. Hence, the only source of this information is the pump manufacturer. Obtain these characteristics from the pump manufacturer or their test data, and plot them on Fig. 46, as shown, for each oil or other liquid handled.

3. *Evaluate the effect of viscosity on pump performance*

Study Fig. 46 to determine the effect of increased liquid viscosity on the performance of the pump. Thus at a given head, say 100 ft (30.5 m), the capacity of the pump decreases as the liquid viscosity increases. At 100-ft (30.5-m) head, this pump has a water capacity of 43.5 gal/min (2.74 L/s), Fig. 46. The pump capacity for the various oils at 100-ft (30.5-m) head is 36 gal/min (2.27 L/s) for 400 SSU; 32 gal/min (2.02 L/s) for 600 SSU; 28 gal/min (1.77 L/s) for 900 SSU; and 26 gal/min (1.64 L/s) for 1000 SSU, respectively. There is a similar reduction in capacity of the pump at the other heads plotted in Fig. 46. Thus, as a general rule, the capacity of a regenerative pump decreases with an increase in liquid viscosity at constant head. Or conversely, at constant capacity, the head developed decreases as the liquid viscosity increases.

Plots of the power input to this pump show that the input power increases as the liquid viscosity increases.

Related Calculations. Use this procedure for a regenerative-type pump handling any liquid—water, oil, kerosene, gasoline, etc. A decrease in the viscosity of a liquid, as compared with the viscosity of water, will produce the opposite effect from that of increased viscosity.

EFFECT OF LIQUID VISCOSITY ON RECIPROCATING-PUMP PERFORMANCE

A direct-acting steam-driven reciprocating pump delivers 100 gal/min (6.31 L/s) of 70°F (21.1°C) water when operating at 50 strokes per minute. How much 2000-SSU crude oil will this pump deliver? How much 125°F (51.7°C) water will this pump deliver?

Calculation Procedure:

1. *Determine the recommended change in pump performance*
Reciprocating pumps of any type—direct-acting or power—having any number of liquid-handling cylinders—one to five or more—are usually rated for maximum delivery when handling 250-SSU liquids or 70°F (21.1°C) water. At higher liquid viscosities or water temperatures, the speed—strokes or rpm—is reduced. Table 1 shows typical recommended speed-correction factors for reciprocating pumps for various liquid viscosities and water temperatures. This table shows that with a liquid viscosity of 2000 SSU the pump speed should be reduced 20 percent. When 125°F (51.7°C) water is handled, the pump speed should be reduced 25 percent, as shown in Table 1.

2. *Compute the delivery of the pump*
The delivery capacity of any reciprocating pump is directly proportional to the number of strokes per minute it makes or to its rpm.

When 2000-SSU oil is used, the pump strokes per minute must be reduced 20 percent, or (50)(0.20) = 10 strokes/min. Hence, the pump speed will be 50 − 10 = 40 strokes/min. Since the delivery is directly proportional to speed, the delivery of 2000-SSU oil = (40/50)(100) = 80 gal/min (5.1 L/s).

When handling 125°F (51.7°C) water, the pump strokes/min must be reduced 25 percent, or (50)(0.5) = 12.5 strokes/min. Hence, the pump speed will be 50.0 − 12.5 = 37.5

TABLE 1. Speed-Correction Factors

Liquid viscosity, SSU	Speed reduction, %	Water temperature °F	Water temperature °C	Speed reduction, %
250	0	70	21.1	0
500	4	80	26.7	9
1000	11	100	37.8	18
2000	20	125	51.7	25
3000	26	150	65.6	29
4000	30	200	93.3	34
5000	35	250	121.1	38

strokes/min. Since the delivery is directly proportional to speed, the delivery of 125°F (51.7°C) water = (37.5/50)(10) = 75 gal/min (4.7 L/s).

Related Calculations. Use this procedure for any type of reciprocating pump handling liquids falling within the range of Table 1. Such liquids include oil, kerosene, gasoline, brine, water, etc.

EFFECT OF VISCOSITY AND DISSOLVED GAS ON ROTARY PUMPS

A rotary pump handles 8000-SSU liquid containing 5 percent entrained gas and 10 percent dissolved gas at a 20-in. (508-mm) Hg pump inlet vacuum. The pump is rated at 1000 gal/min (63.1 L/s) when handling gas-free liquids at viscosities less than 600 SSU. What is the output of this pump without slip? With 10 percent slip?

Calculation Procedure:

1. *Compute the required speed reduction of the pump*

When the liquid viscosity exceeds 600 SSU, many pump manufacturers recommend that the speed of a rotary pump be reduced to permit operation without excessive noise or vibration. The speed reduction usually recommended is shown in Table 2.

With this pump handling 8000-SSU liquid, a speed reduction of 40 percent is necessary, as shown in Table 2. Since the capacity of a rotary pump varies directly with its speed, the output of this pump when handling 8000-SSU liquid = (1000 gal/min) × (1.0 − 0.40) = 600 gal/min (37.9 L/s).

2. *Compute the effect of gas on the pump output*

Entrained or dissolved gas reduces the output of a rotary pump, as shown in Table 3. The gas in the liquid expands when the inlet pressure of the pump is below atmospheric and the gas occupies part of the pump chamber, reducing the liquid capacity.

TABLE 2. Rotary Pump Speed Reduction for Various Liquid Viscosities

Liquid viscosity, SSU	Speed reduction, percent of rated pump speed
600	2
800	6
1,000	10
1,500	12
2,000	14
4,000	20
6,000	30
8,000	40
10,000	50
20,000	55
30,000	57
40,000	60

TABLE 3. Effect of Entrained or Dissolved Gas on the Liquid Displacement of Rotary Pumps (liquid displacement: percent of displacement)

Vacuum at pump inlet, in Hg (mmHg)	Gas entrainment					Gas solubility					Gas entrainment and gas solubility combined				
	1%	2%	3%	4%	5%	2%	4%	6%	8%	10%	1% 2%	2% 4%	3% 6%	4% 8%	5% 10%
5 (127)	99	97½	96½	95	93½	99½	99	98½	97	97½	98½	96½	96	92	91
10 (254)	98½	97¼	95½	94	92	99	97½	97	95	95	97½	95	90	90	88¼
15 (381)	98	96½	94½	92½	90½	97	96	94	92	90½	96	93	89½	86½	83¼
20 (508)	97½	94½	92	89	86½	96	92	89	86	83	94	88	83	78	74
25 (635)	94	89	84	79	75½	90	83	76½	71	66	85½	75½	68	61	55

For example, with 5 percent gas entrainment at 15 in Hg (381 mmHg) vacuum, the liquid displacement will be 90½ percent of the pump displacement, neglecting slip, or with 10 percent dissolved gas liquid displacement will be 90½ percent of the pump displacement; and with 5 percent entrained gas combined with 10 percent dissolved gas, the liquid displacement will be 83¼ percent of pump replacement.

Source: Courtesy of Kinney Mfg. Div., The New York Air Brake Co.

With a 20-in.(508-mm) Hg inlet vacuum, 5 percent entrained gas, and 10 percent dissolved gas, Table 3 shows that the liquid displacement is 74 percent of the rated displacement. Thus, the output of the pump when handling this viscous, gas-containing liquid will be (600 gal/min)(0.74) = 444 gal/min (28.0 L/s) without slip.

3. *Compute the effect of slip on the pump output*

Slip reduces rotary-pump output in direct proportion to the slip. Thus, with 10 percent slip, the output of this pump = (444 gal/min)(1.0 − 0.10) = 369.6 gal/min (23.3 L/s).

Related Calculations. Use this procedure for any type of rotary pump—gear, lobe, screw, swinging-vane, sliding-vane, or shuttle-block, handling any clear, viscous liquid. Where the liquid is gas-free, apply only the viscosity correction. Where the liquid viscosity is less than 600 SSU but the liquid contains gas or air, apply the entrained or dissolved gas correction, or both corrections.

SELECTION OF MATERIALS FOR PUMP PARTS

Select suitable materials for the principal parts of a pump handling cold ethylene chloride. Use the Hydraulic Institute recommendation for materials of construction.

Calculation Procedure:

1. *Determine which materials are suitable for this pump*

Refer to the data section of the Hydraulic Institute *Standards*. This section contains a tabulation of hundreds of liquids and the pump construction materials that have been successfully used to handle each liquid.

The table shows that for cold ethylene chloride having a specific gravity of 1.28, an all-bronze pump is satisfactory. In lieu of an all-bronze pump, the principal parts of the pump—casing, impeller, cylinder, and shaft—can be made of one of the following materials: austenitic steels (low-carbon 18-8; 18-8/Mo; highly alloyed stainless); nickel-base alloys containing chromium, molybdenum, and other elements, and usually less than 20 percent iron; or nickel-copper alloy (Monel metal). The order of listing in the *Standards* does not necessarily indicate relative superiority, since certain factors predominating in one instance may be sufficiently overshadowed in others to reverse the arrangement.

2. *Choose the most economical pump*

Use the methods of earlier calculation procedures to select the most economical pump for the installation. Where the corrosion resistance of two or more pumps is equal, the standard pump, in this instance an all-bronze unit, will be the most economical.

Related Calculations. Use this procedure to select the materials of construction for any class of pump—centrifugal, rotary, or reciprocating—in any type of service—power, process, marine, or commercial. Be certain to use the latest edition of the Hydraulic Institute *Standards*, because the recommended materials may change from one edition to the next.

SIZING A HYDROPNEUMATIC STORAGE TANK

A 200-gal/min (12.6-L/s) water pump serves a pumping system. Determine the capacity required for a hydropneumatic tank to serve this system if the allowable high pressure in the tank and system is 60 lb/sq.in. (gage) (413.6 kPa) and the allowable low pressure is 30 lb/sq.in. (gage) (206.8 kPa). How many starts per hour will the pump make if the system draws 3000 gal/min (189.3 L/s) from the tank?

Calculation Procedure:

1. ***Compute the required tank capacity***
In the usual hydropneumatic system, a storage-tank capacity in gal of 10 times the pump capacity in gal/min is used, if this capacity produces a moderate running time for the pump. Thus, this system would have a tank capacity of (10)(200) = 2000 gal (7570.8 L).

2. ***Compute the quantity of liquid withdrawn per cycle***
For any hydropneumatic tank the withdrawal, expressed as the number of gallons (liters) withdrawn per cycle, is given by $W = (v_L - v_H)/C$, where v_L = air volume in tank at the lower pressure, ft^3 (m^3); v_H = volume of air in tank at higher pressure, ft^3 (m^3); C = conversion factor to convert ft^3 (m^3) to gallons (liters), as given below.

Compute V_L and V_H using the gas law for v_H and either the gas law or the reserve percentage for v_L. Thus, for v_H, the gas law gives $v_H = p_L \mathrm{v_L}/p_H$, where p_L = lower air pressure in tank, lb/sq.in. (abs) (kPa); p_H = higher air pressure in tank lb/sq.in. (abs) (kPa); other symbols as before.

In most hydropneumatic tanks a liquid reserve of 10 to 20 percent of the total tank volume is kept in the tank to prevent the tank from running dry and damaging the pump. Assuming a 10 percent reserve for this tank, $v_L = 0.1\ V$, where V = tank volume in ft^3 (m^3). Since a 2000-gal (7570-L) tank is being used, the volume of the tank is 2000/7.481 ft^3/gal = 267.3 ft^3 (7.6 m^3). With the 10 percent reserve at the 44.7 lb/sq.in. (abs) (308.2-kPa) lower pressure, $v_L = 0.9\ (267.3) = 240.6$ ft^3 (6.3 m^3), where $0.9 = V - 0.1\ V$.

At the higher pressure in the tank, 74.7 lb/sq.in. (abs) (514.9 kPa), the volume of the air will be, from the gas law, $v_H = p_L \mathrm{v_L}/p_H = 44.7\ (240.6)/74.7 = 143.9$ ft^3 (4.1 m^3). Hence, during withdrawal, the volume of liquid removed from the tank will be $W_g = (240.6 - 143.9)/0.1337 = 723.3$ gal (2738 L). In this relation the constant converts from cubic feet to gallons and is 0.1337. To convert from cubic meters to liters, use the constant 1000 in the denominator.

3. ***Compute the pump running time***
The pump has a capacity of 200 gal/min (12.6 L/s). Therefore, it will take 723/200 = 3.6 min to replace the withdrawn liquid. To supply 3000 gal/h (11,355 L/h) to the system, the pump must start 3000/723 = 4.1, or 5 times per hour. This is acceptable because a system in which the pump starts six or fewer times per hour is generally thought satisfactory.

Where the pump capacity is insufficient to supply the system demand for short periods, use a smaller reserve. Compute the running time using the equations in steps 2 and 3. Where a larger reserve is used—say 20 percent—use the value 0.8 in the equations in step 2. For a 30 percent reserve, the value would be 0.70, and so on.

Related Calculations. Use this procedure for any liquid system having a hydropneumatic tank—well drinking water, marine, industrial, or process.

USING CENTRIFUGAL PUMPS AS HYDRAULIC TURBINES

Select a centrifugal pump to serve as a hydraulic turbine power source for a 1500-gal/min (5677.5-L/min) flow rate with 1290 ft (393.1 m) of head. The power application requires a 3600-r/min speed, the specific gravity of the liquid is 0.52, and the total available exhaust head is 20 ft (6.1 m). Analyze the cavitation potential and operating characteristics at an 80 percent flow rate.

Calculation Procedure:

1. *Choose the number of stages for the pump*

Search of typical centrifugal-pump data shows that a head of 1290 ft (393.1 m) is too large for a single-stage pump of conventional design. Hence, a two-stage pump will be the preliminary choice for this application. The two-stage pump chosen will have a design head of 645 ft (196.6 m) per stage.

2. *Compute the specific speed of the pump chosen*

Use the relation N_s = pump $rpm(Q)^{0.5}/H^{0.75}$, where N_s = specific speed of the pump; *rpm* = r/min of pump shaft; Q = pump capacity or flow rate, gal/min; H = pump head per stage, ft. Substituting, we get $N_s = 3600(1500)^{0.5}/(645)^{0.75} = 1090$. Note that the specific speed value is the same regardless of the system of units used—USCS or SI.

3. *Convert turbine design conditions to pump design conditions*

To convert from turbine design conditions to pump design conditions, use the pump manufacturer's conversion factors that relate turbine best efficiency point (bep) performance with pump bep performance. Typically, as specific speed N_s varies from 500 to 2800, these bep factors generally vary as follows: the conversion factor for capacity (gal/min or L/min) C_Q, from 2.2 to 1.1; the conversion factor for head (ft or m) C_H, from 2.2 to 1.1; the conversion factor for efficiency C_E, from 0.92 to 0.99. Applying these conversion factors to the turbine design conditions yields the pump design conditions sought.

At the specific speed for this pump, the values of these conversion factors are determined from the manufacturer to be $C_Q = 1.24$; $C_H = 1.42$; $C_E = 0.967$.

Given these conversion factors, the turbine design conditions can be converted to the pump design conditions thus: $Q_p = Q_t/C_Q$, where Q_p = pump capacity or flow rate, gal/min or L/min; Q_t = turbine capacity or flow rate in the same units; other symbols are as given earlier. Substituting gives $Q_p = 1500/1.24 = 1210$ gal/min (4580 L/min).

Likewise, the pump discharge head, in feet of liquid handled, is $H_p = H_t/C_H$. So $H_p = 645/1.42 = 454$ ft (138.4 m).

4. *Select a suitable pump for the operating conditions*

Once the pump capacity, head, and rpm are known, a pump having its best bep at these conditions can be selected. Searching a set of pump characteristic curves and capacity tables shows that a two-stage 4-in. (10-cm) unit with an efficiency of 77 percent would be suitable.

5. *Estimate the turbine horsepower developed*

To predict the developed hp, convert the pump efficiency to turbine efficiency. Use the conversion factor developed above. Or, the turbine efficiency $E_t = E_p\,C_E = (0.77)(0.967) = 0.745$, or 74.5 percent.

With the turbine efficiency known, the output brake horsepower can be found from bhp $= Q_tH_tE_ts/3960$, where s = fluid specific gravity; other symbols as before. Substituting, we get bhp $= 1500(1290)(0.745)(0.52)/3960 = 198$ hp (141 kW).

6. *Determine the cavitation potential of this pump*

Just as pumping requires a minimum net positive suction head, turbine duty requires a net positive exhaust head. The relation between the total required exhaust head (TREH) and turbine head per stage is the cavitation constant σ_r = TREH/H. Figure 5 shows σ_r vs. N_s

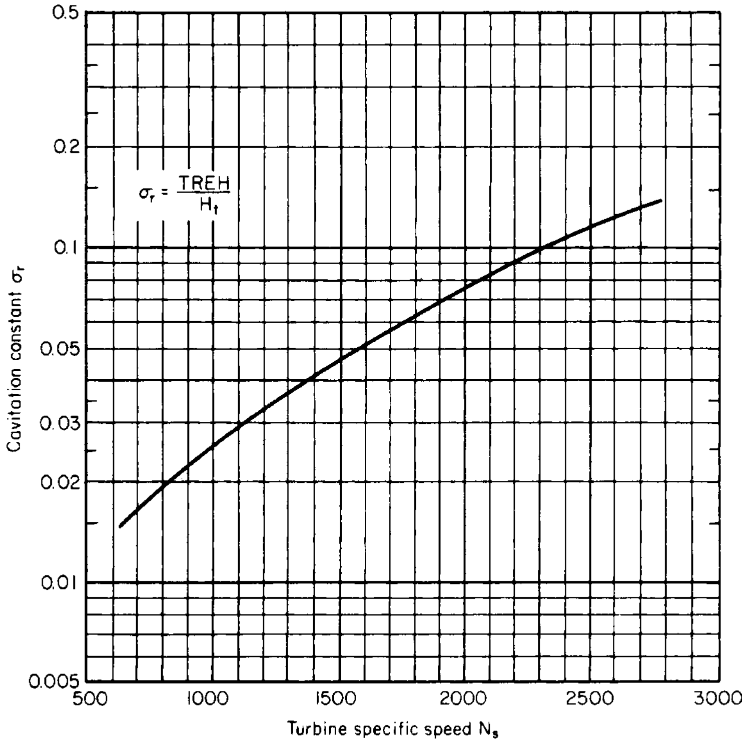


FIGURE 47. Cavitation constant for hydraulic turbines. (*Chemical Engineering.*)

for hydraulic turbines. Although a pump used as a turbine will not have exactly the same relationship, this curve provides a good estimate of σ_r for turbine duty.

To prevent cavitation, the total available exhaust head (TAEH) must be greater than the TREH. In this installation, N_s = 1090 and TAEH = 20 ft (6.1 m). From Fig. 47, σ_r = 0.028 and TREH = 0.028(645) = 18.1 ft (5.5 m). Because TAEH > TREH, there is enough exhaust head to prevent cavitation.

7. *Determine the turbine performance at 80 percent flow rate*

In many cases, pump manufacturers treat conversion factors as proprietary information. When this occurs, the performance of the turbine under different operating conditions can be predicted from the general curves in Figs. 48 and 49.

At the 80 percent flow rate for the turbine, or 1200 gal/min (4542 L/min), the operating point is 80 percent of bep capacity. For a specific speed of 1090, as before, the percentages of bep head and efficiency are shown in Figs. 48 and 49: 79.5 percent of bep head and percent of bep efficiency. To find the actual performance, multiply by the bep values, or, H_t = 0.795(1290) = 1025 ft (393.1 m); E_t = 0.91(74.5) = 67.8 percent.

The bhp at the new operating condition is then bhp = 1200 (1025)(0.678)(0.52)/3960 = 110 hp (82.1 kW).

In a similar way, the constant-head curves in Figs. 50 and 51 predict turbine performance at different speeds. For example, speed is 80 percent of bep speed at 2880 r/min. For a specific speed of 1090, the percentages of bep capacity, efficiency, and power are

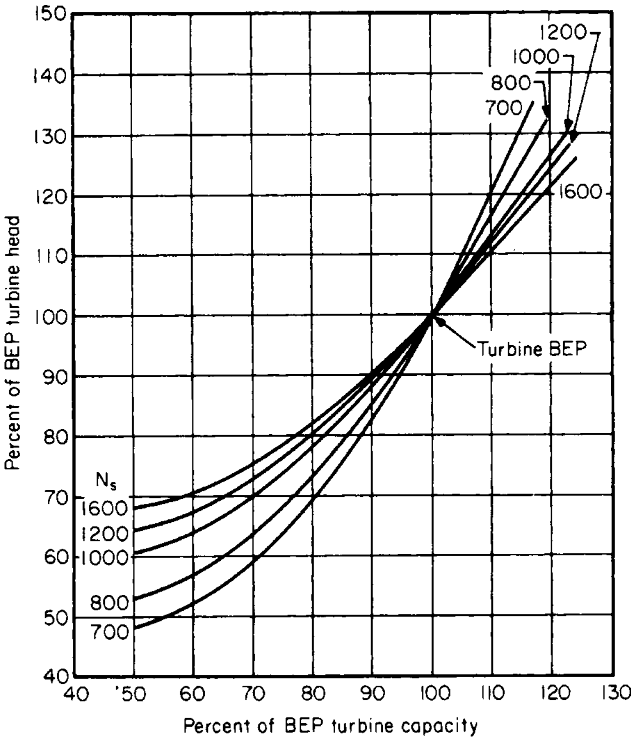


FIGURE 48. Constant-speed curves for turbine duty. (*Chemical Engineering.*)

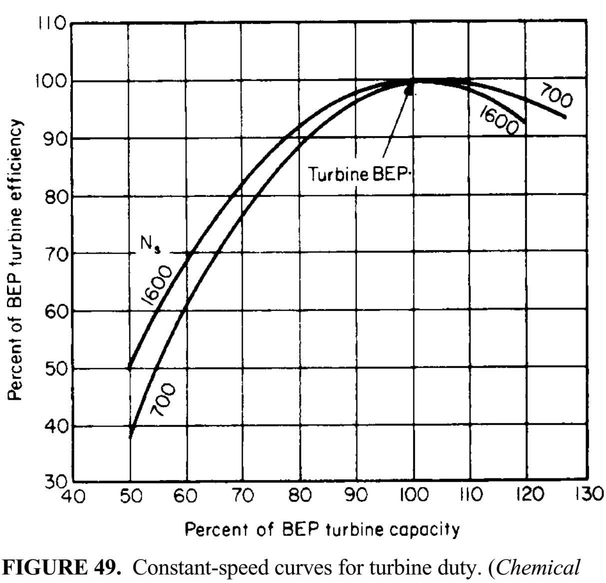


FIGURE 49. Constant-speed curves for turbine duty. (*Chemical Engineering.*)

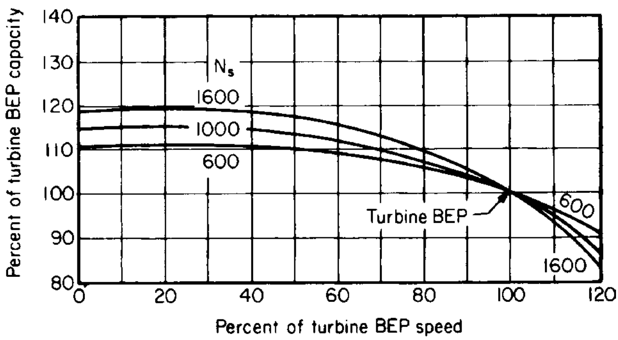


FIGURE 50. Constant-head curves for turbine duty. (*Chemical Engineering.*)

107 percent of the capacity, 94 percent of the efficiency, and 108 percent of the bhp. To get the actual performance, convert as before: Q_t = 107(1500) = 1610 gal/min (6094 L/min); E_t = 0.94(74.5) = 70.0 percent; bhp = 1.08(189) = 206 hp (153.7 kW).

Note that the bhp in this last instance is higher than the bhp at the best efficiency point. Thus more horsepower can be obtained from a given unit by reducing the speed and increasing the flow rate. When the speed is fixed, more bhp cannot be obtained from the unit, but it may be possible to select a smaller pump for the same application.

Related Calculations. Use this general procedure for choosing a centrifugal pump to drive—as a hydraulic turbine—another pump, a fan, a generator, or a compressor, where high-pressure liquid is available as a source of power. Because pumps are designed

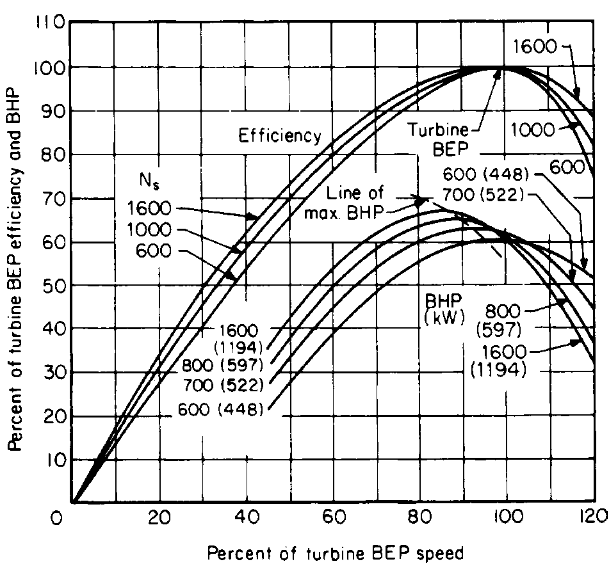


FIGURE 51. Constant-head curves for turbine duty. (*Chemical Engineering.*)

as fluid movers, they may be less efficient as hydraulic turbines than equipment designed for that purpose. Steam turbines and electric motors are more economical when steam or electricity is available.

But using a pump as a turbine can pay off in remote locations where steam or electric power would require additional wiring or piping, in hazardous locations that require non-sparking equipment, where energy may be recovered from a stream that otherwise would be throttled, and when a radial-flow centrifugal pump is immediately available but a hydraulic turbine is not.

In the most common situation, there is a liquid stream with fixed head and flow rate and an application requiring a fixed rpm; these are the turbine design conditions. The objective is to pick a pump with a turbine bep at these conditions. With performance curves such as Fig. 46, turbine design conditions can be converted to pump design conditions. Then you select from a manufacturer's catalog a model that has its pump bep at those values.

The most common error in pump selection is using the turbine design conditions in choosing a pump from a catalog. Because catalog performance curves describe pump duty, not turbine duty, the result is an oversized unit that fails to work properly.

This procedure is the work of Fred Buse, Chief Engineer, Standard Pump Aldrich Division of Ingersoll-Rand Co., as reported in *Chemical Engineering* magazine.

SIZING CENTRIFUGAL-PUMP IMPELLERS FOR SAFETY SERVICE

Determine the impeller size of a centrifugal pump that will provide a safe continuous-recirculation flow to prevent the pump from overheating at shutoff. The pump delivers 320 gal/min (20.2 L/s) at an operating head of 450 ft (137.2 m). The inlet water temperature is 220°F (104.4°C), and the system has an NPSH of 5 ft (1.5 m). Pump performance curves and the system-head characteristic curve for the discharge flow (without recirculation) are shown in Fig. 53, and the piping layout is shown in Fig. 54. The brake horsepower (bhp) of an 11-in. (27.9-cm) and an 11.5-in. (29.2-cm) impeller at shutoff is 53 and 60, respectively. Determine the permissible water temperature rise for this pump.

Calculation Procedure:

1. *Compute the actual temperature rise of the water in the pump*

Use the relation $P_0 = P_v + P_{\mathrm{NPSH}}$, where P_0 = pressure corresponding to the actual liquid temperature in the pump during operation, lb/sq.in. (abs) (kPa); P_v = vapor pressure in the pump at the inlet water temperature, lb/sq.in. (abs) (kPa); P_{NPSH} = pressure created by the net positive suction head on the pumps, lb/sq.in. (abs) (kPa). The head in feet (meters) must be converted to lb/sq.in. (abs) (kPa) by the relation lb/sq.in. (abs) = (NPSH, ft) (liquid density at the pumping temperature, lb/ft^3)/(144 sq.in./sq.ft.). Substituting yields P_0 = 17.2 lb/sq.in. (abs) + 5(59.6)/144 = 19.3 lb/sq.in. (abs) (133.1 kPa).

Using the steam tables, find the saturation temperature T_s corresponding to this absolute pressure as T_s = 226.1°F (107.8°C). Then the permissible temperature rise is $T_p = T_s - T_{op}$, where T_{op} = water temperature in the pump inlet. Or, T_p = 226.1 − 220 = 6.1°F (3.4°C).

2. *Compute the recirculation flow rate at the shutoff head*

From the pump characteristic curve with recirculation, Fig. 55, the continuous-recirculation flow Q_B for an 11.5-in. (29.2-cm) impeller at an operating head of 450 ft (137.2 m) is

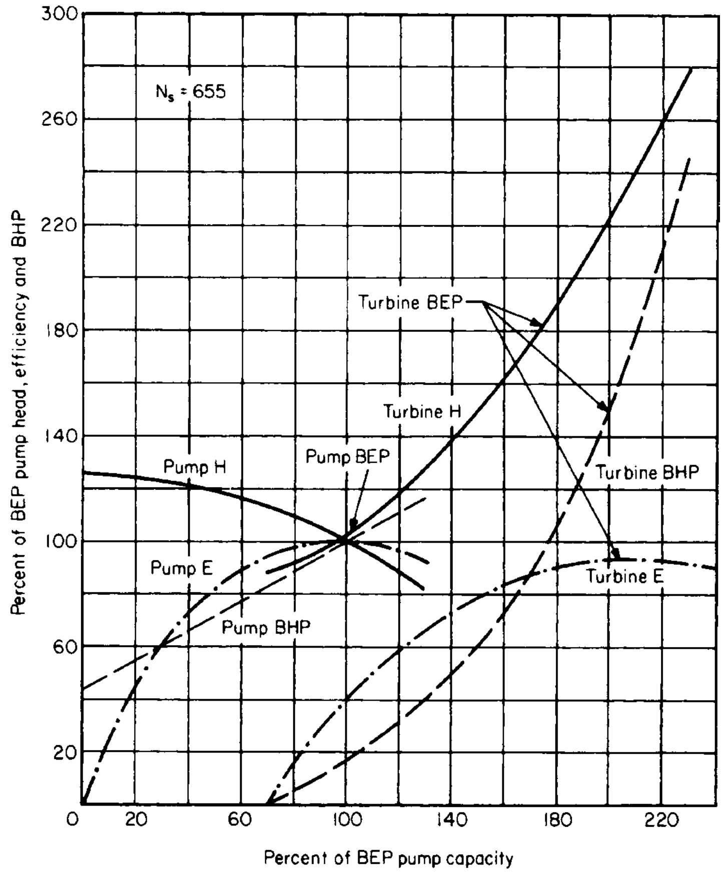


FIGURE 52. Performance of a pump at constant speed in pump duty and turbine duty. (*Chemical Engineering.*)

48.6 gal/min (177.1 L/min). Find the continuous-recirculation flow at shutoff head H_s ft (m) of 540 ft (164.6 m) from $Q_s = Q_B(H_s/H_{op})^{0.5}$, where H_{op} = operating head, ft (m). Or Q_s = 48.6(540/450) = 53.2 gal/min (201.4 L/min).

3. *Find the minimum safe flow for this pump*

The minimum safe flow, lb/h, is given by $w_{min} = 2545bhp/[C_pT_p + (1.285 \times 10^{-3})H_s]$, where C_p = specific head of the water; other symbols as before. Substituting, we find $w_{min} = 2545(60)/[1.0(6.1) + (1.285 \times 10^{-3})(540)]$ = 22,476 lb/h (2.83 kg/s). Converting to gal/min yields $Q_{min} = w_{min}/[(\text{ft}^3/\text{h})(\text{gal/min})(\text{lb/ft}^3)]$ for the water flowing through the pump. Or, Q_{min} = 22,476/[(8.021)(59.6)] = 47.1 gal/min (178.3 L/min).

4. *Compare the shutoff recirculation flow with the safe recirculation flow*

Since the shutoff recirculation flow Q_s = 53.2 gal/min (201.4 L/min) is greater than Q_{min} = 47.1 gal/min (178.3 L/min), the 11.5-in. (29.2-cm) impeller is adequate to provide safe continuous recirculation. An 11.25-in. (28.6-cm) impeller would not be adequate because Q_{min} = 45 gal/min (170.3 L/min) and Q_s = 25.6 gal/min (96.9 L/min).

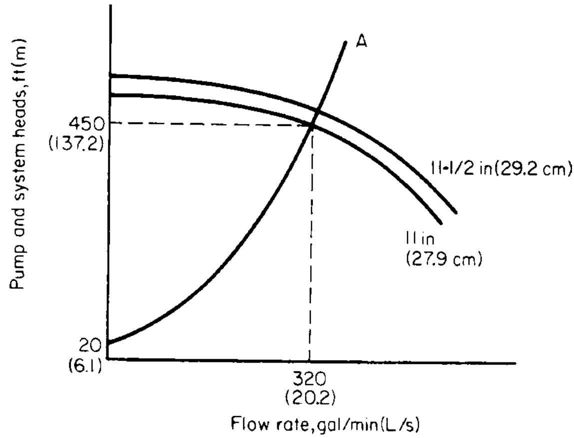


FIGURE 53. System-head curves without recirculation flow. (*Chemical Engineering.*)

Related Calculations. Safety-service pumps are those used for standby service in a variety of industrial plants serving the chemical, petroleum, plastics, aircraft, auto, marine, manufacturing, and similar businesses. Such pumps may be used for water supply, fire protection, boiler feed, condenser cooling, and related tasks. In such systems the pump is usually oversized and has a recirculation loop piped in to prevent overheating by maintaining a minimum safe flow. Figure 54 shows a schematic of such a system. Recirculation is controlled by a properly sized orifice rather than by valves because an orifice is less expensive and highly reliable.

The general procedure for sizing centrifugal pumps for safety service, using the symbols given earlier, is this: (1) Select a pump that will deliver the desired flow Q_A, using

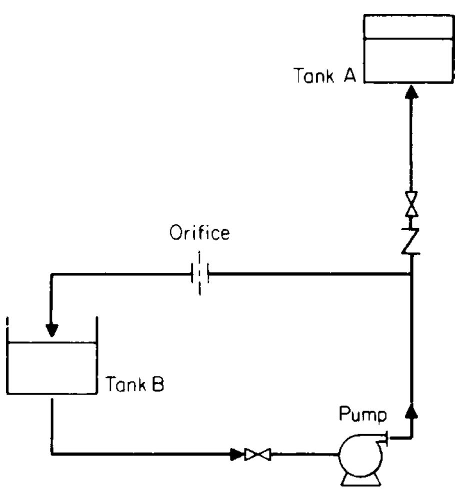


FIGURE 54. Pumping system with a continuous-recirculation line. (*Chemical Engineering.*)

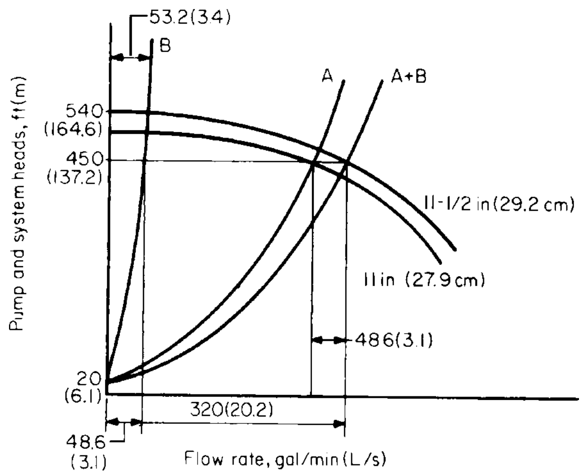


FIGURE 55. System-head curves with recirculation flow. (*Chemical Engineering.*)

the head-capacity characteristic curves of the pump and system. (2) Choose the next larger diameter pump impeller to maintain a discharge flow of Q_A to tank *A*, Fig. 54, and a recirculation flow Q_B to tank *B*, Fig. 54. (3) Compute the recirculation flow Q_s at the pump shutoff point from $Q_s = Q_B(H_s/H_{op})^{0.5}$. (4) Calculate the minimum safe flow $Q_{\min}$ for the pump with the larger impeller diameter. (5) Compare the recirculation flow Q_s at the pump shutoff point with the minimum safe flow $Q_{\min}$. If $Q_s \geq Q_{\min}$, the selection process has been completed. If $Q_s < Q_{\min}$, choose the next larger size impeller and repeat steps 3, 4, and 5 above until the impeller size that will provide the minimum safe recirculation flow is determined.

This procedure is the work of Mileta Mikasinovic and Patrick C. Tung, design engineers, Ontario Hydro, as reported in *Chemical Engineering* magazine.

PUMP CHOICE TO REDUCE ENERGY CONSUMPTION AND LOSS

Choose an energy-efficient pump to handle 1000 gal/min (3800 L/min) of water at 60°F (15.6°C) at a total head of 150 ft (45.5 m). A readily commercially available pump is preferred for this application.

Calculation Procedure:

1. *Compute the pump horsepower required*

For any pump, $bhp_i = (gpm)(H_t)(s)/3960e$, where bhp_i = input brake (motor) horsepower to the pump; H_t = total head on the pump, ft; s = specific gravity of the liquid handled; e = hydraulic efficiency of the pump. For this application where $s = 1.0$ and a hydraulic efficiency of 70 percent can be safely assumed, $bhp_i = (1000)(150)(1)/(3960)(0.70) = 54.1$ bhp (40.3 kW).

2. *Choose the most energy-efficient pump*

Use Fig. 56, entering at the bottom at 1000 gal/min (3800 L/min) and projecting vertically upward to a total head of 150 ft (45.5 m). The resulting intersection is within area 1, showing from Table 4 that a single-stage 3500-r/min electric-motor-driven pump would be the most energy-efficient.

Related Calculations. The procedure given here can be used for pumps in a variety of applications—chemical, petroleum, commercial, industrial, marine, aeronautical, air-conditioning, cooling-water, etc., where the capacity varies from 10 to 1,000,000 gal/min (38 to 3,800,000 L/min) and the head varies from 10 to 10,000 ft (3 to 3300 m). Figure 14 is based primarily on the characteristic of pump specific speed $N_s = NQ^2/H^{3/4}$, where N = pump rotating speed, r/min; Q = capacity, gal/min (L/min); H = total head, ft (m).

When N_s is less than 1000, the operating efficiency of single-stage centrifugal pumps falls off dramatically; then either multistage or higher-speed pumps offer the best efficiency.

Area 1 of Fig. 56 is the densest, crowded both with pumps operating at 1750 and 3500 r/min, because years ago, 3500-r/min pumps were not thought to be as durable as 1750-r/min ones. Since the adoption of the AVS standard in 1960 (superseded by ANSI B73.1), pumps with stiffer shafts have been proved reliable.

Also responsible for many 1750-r/min pumps in area 1 has been the impression that the higher (3500-r/min) speed causes pumps to wear out faster. However, because impeller tip speed is the same at both 3500 and 1750 r/min [as, for example, a 6-in. (15-cm) impeller at 3500 r/min and a 12-in. (30-cm) one at 1740 r/min], so is the fluid velocity, and so should be the erosion of metal surface. Another reason for not limiting operating

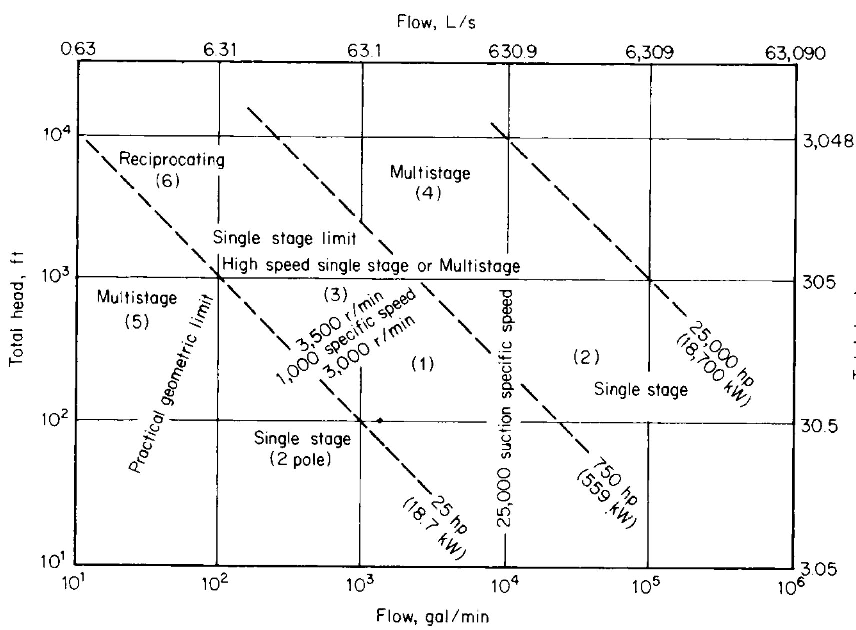


FIGURE 56. Selection guide is based mainly on specific speed, which indicates impeller geometry. (*Chemical Engineering.*)

TABLE 4. Type of Pump for Highest Energy Efficiency*

Area 1:	Single-stage, 3500 r/min
Area 2:	Single-stage, 1750 r/min or lower
Area 3:	Single-stage, above 3500 r/min, or multistage, 3500 r/min
Area 4:	Multistage
Area 5:	Multistage
Area 6:	Reciprocating

*Includes ANSI B73.1 standards, see area number in Fig. 56.

speed is that improved impeller inlet design allows operation at 3500 r/min to capacities of 5000 gal/min (19,000 L/min) and higher.

Choice of operating speed also may be indirectly limited by specifications pertaining to suction performance, such as that fixing the top suction specific speed *S* directly or indirectly by choice of the sigma constant or by reliance on Hydraulic Institute charts.

Values of *S* below 8000 to 10,000 have long been accepted for avoiding cavitation. However, since the development of the inducer, *S* values in the range of 20,000 to 25,000 have become commonplace, and values as high as 50,000 have become practical.

The sigma constant, which relates NPSH to total head, is little used today, and Hydraulic Institute charts (which are being revised) are conservative.

In light of today's designs and materials, past restrictions resulting from suction performance limitations should be reevaluated or eliminated entirely.

Even if the most efficient pump has been selected, there are a number of circumstances in which it may not operate at peak efficiency. Today's cost of energy has made these considerations more important.

A centrifugal pump, being a hydrodynamic machine, is designed for a single peak operating-point capacity and total head. Operation at other than this best efficiency point (bep) reduces efficiency. Specifications now should account for such factors as these:

1. A need for a larger number of smaller pumps. When a process operates over a wide range of capacities, as many do, pumps will often work at less than full capacity, hence at lower efficiency. This can be avoided by installing two or three pumps in parallel, in place of a single large one, so that one of the smaller pumps can handle the flow when operations are at a low rate.

2. Allowance for present capacity. Pump systems are frequently designed for full flow at some time in the future. Before this time arrives, the pumps will operate far from their best efficiency points. Even if this interim period lasts only 2 or 3 years, it may be more economical to install a smaller pump initially and to replace it later with a full-capacity one.

3. Inefficient impeller size. Some specifications call for pump impeller diameter to be no larger than 90 or 95 percent of the size that a pump could take, so as to provide reserve head. If this reserve is used only 5 percent of the time, all such pumps will be operating at less than full efficiency most of the time.

4. Advantages of allowing operation to the right of the best efficiency point. Some specifications, the result of such thinking as that which provides reserve head, prohibit the selection of pumps that would operate to the right of the best efficiency point. This

eliminates half of the pumps that might be selected and results in oversized pumps operating at lower efficiency.

This procedure is the work of John H. Doolin, Director of Product Development, Worthington Pumps, Inc., as reported in *Chemical Engineering* magazine.

SMALL HYDRO POWER CONSIDERATIONS AND ANALYSIS

A city is considering a small hydro power installation to save fossil fuel. To obtain the savings, the following steps will be taken: refurbish an existing dam, install new turbines, operate the generating plant. Outline the considerations a designer must weigh before undertaking the actual construction of such a plant.

Calculation Procedure:

1. *Analyze the available head*
Most small hydro power sites today will have a head of less than 50 ft (15.2 m) between the high-water level and tail-water level, Fig. 57. The power-generating capacity will usually be 25 MW or less.

2. *Relate absolute head to water flow rate*
Because heads across the turbine in small hydro installations are often low in magnitude, the tail-water level is important in assessing the possibilities of a given site. At high-water flows, tail-water levels are often high enough to reduce turbine output, Fig. 58*a*. At some sites, the available head at high flow is extremely low, Fig. 58*b*.

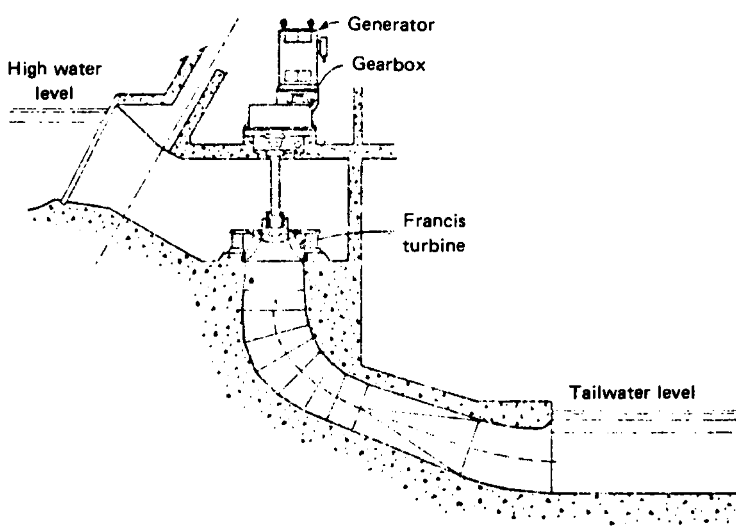


FIGURE 57. Vertical Francis turbine in open pit was adapted to 8-m head in an existing Norwegian dam. (*Power.*)

The actual power output from a hydro station is $P = HQwe/550$, where P = horsepower output; H = head across turbine, ft; Q = water flow rate, ft^3/s; w = weight of water, lb/ft^3; e = turbine efficiency. Substituting in this equation for the plant shown in Fig. 58*b*, for flow rates of 500 and 1500 m^3/s, we see that a tripling of the water flow rate increases the power output by only 38.7 percent, while the absolute head drops 53.8 percent (from 3.9 to 1.8 m). This is why the tail-water level is so important in small hydro installations.

Figure 58*c* shows how station costs can rise as head decreases. These costs were estimated by the Department of Energy (DOE) for a number of small hydro power installations. Figure 58 shows that station cost is more sensitive to head than to power capacity, according to DOE estimates. And the prohibitive costs for developing a completely new small hydro site mean that nearly all work will be at existing dams. Hence, any water exploitation for power must not encroach seriously on present customs, rights, and usages of the water. This holds for both upstream and downstream conditions.

3. *Outline machinery choice considerations*

Small-turbine manufacturers, heeding the new needs, are producing a good range of semistandard designs that will match any site needs in regard to head, capacity, and excavation restrictions.

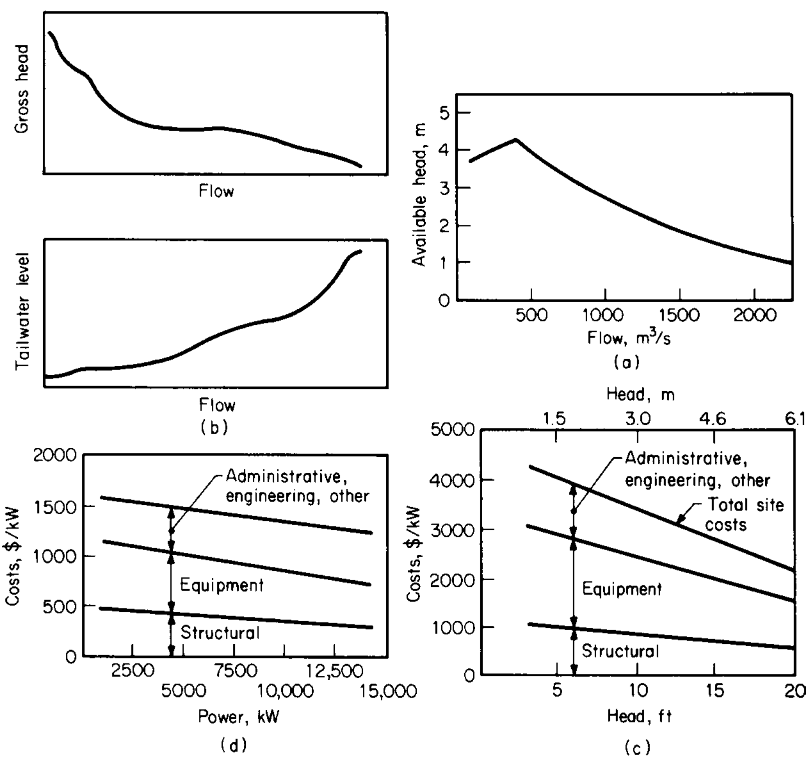


FIGURE 58. (*a*) Rising tail-water level in small hydro projects can seriously curtail potential. (*b*) Anderson-Cottonwood dam head dwindles after a peak at low flow. (*c*) Low heads drive DOE estimates up. (*d*) Linear regression curves represent DOE estimates of costs of small sites. (*Power.*)

The Francis turbine, Fig. 57, is a good example of such designs. A horizontal-shaft Francis turbine may be a better choice for some small projects because of lower civil-engineering costs and compatibility with standard generators.

Efficiency of small turbines is a big factor in station design. The problem of full-load versus part-load efficiency, Fig. 59, must be considered. If several turbines can fit the site needs, then good part-load efficiency is possible by load sharing.

Fitting new machinery to an existing site requires ingenuity. If enough of the old powerhouse is left, the same setup for number and type of turbines might be used. In other installations the powerhouse may be absent, badly deteriorated, or totally unsuitable. Then river-flow studies should be made to determine which of the new semistandard machines will best fit the conditions.

Personnel costs are extremely important in small hydro projects. Probably very few small hydro projects centered on redevelopment of old sites can carry the burden of workers in constant attendance. Hence, personnel costs should be given close attention.

Tube and bulb turbines, with horizontal or nearly horizontal shafts, are one way to solve the problem of fitting turbines into a site without heavy excavation or civil engineering works. Several standard and semistandard models are available.

In low head work, the turbine is usually low-speed, far below the speed of small generators. A speed-increasing gear box is therefore required. A simple helical-gear unit is satisfactory for vertical-shaft and horizontal-shaft turbines. Where a vertical turbine drives a horizontal generator, a right-angle box makes the turn in the power flow.

Governing and control equipment is not a serious problem for small hydro plants.

Related Calculations. Most small hydro projects are justified on the basis of continuing inflation, which will make the savings they produce more valuable as time passes. Although this practice is questioned by some people, the recent history of inflation seems to justify the approach.

As fossil-fuel prices increase, small hydro installations will become more feasible. However, the considerations mentioned in this procedure should be given full weight before proceeding with the final design of any plant. The data in this procedure were drawn from an ASME meeting on the subject with information from papers, panels, and discussion summarized by William O'Keefe, Senior Editor, *Power* magazine, in an article in that publication.

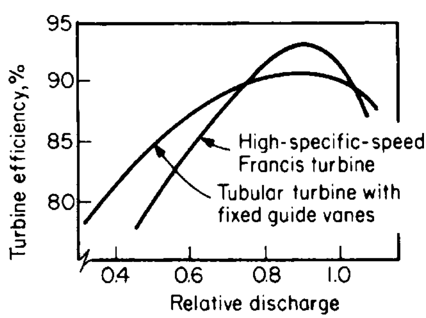


FIGURE 59. Steep Francis-turbine efficiency falloff frequently makes multiple units advisable.

"CLEAN" ENERGY FROM SMALL-SCALE HYDRO SITES

A newly discovered hydro site provides a potential head of 65 ft (20 m). An output of 10,000 kW (10 MW) is required to justify use of the site. Select suitable equipment for this installation based on the available head and the required power output.

Calculation Procedure:

1. *Determine the type of hydraulic turbine suitable for this site*
Enter Fig. 60 on the left at the available head, 65 ft (20 m), and project to the right to intersect the vertical projection from the required turbine output of 10,000 kW (10 MW). These two lines intersect in the *standardized tubular unit* region. Hence, such a hydroturbine will be tentatively chosen for this site.

2. *Check the suitability of the chosen unit*
Enter Table 5 at the top at the operating head range of 65 ft (20 m) and project across to the left to find that a tubular-type hydraulic turbine with fixed blades and adjustable gates will produce 0.25 to 15 MW of power at 55 to 150 percent of rated head. These ranges are within the requirements of this installation. Hence, the type of unit indicated by Fig. 60 is suitable for this hydro site.

Related Calculations. Passage of legislation requiring utilities to buy electric power from qualified site developers is leading to strong growth of both site development and equipment suitable for small-scale hydro plants. Environmental concerns over fossil-fuel-fired and nuclear generating plants make hydro power more attractive. Hydro plants, in general, do not pollute the air, do not take part in the acid-rain cycle, are usually remote from populated areas, and run for up to 50 years with low maintenance and repair costs. Environmentalists rate hydro power as "clean" energy available with little, or no, pollution of the environment.

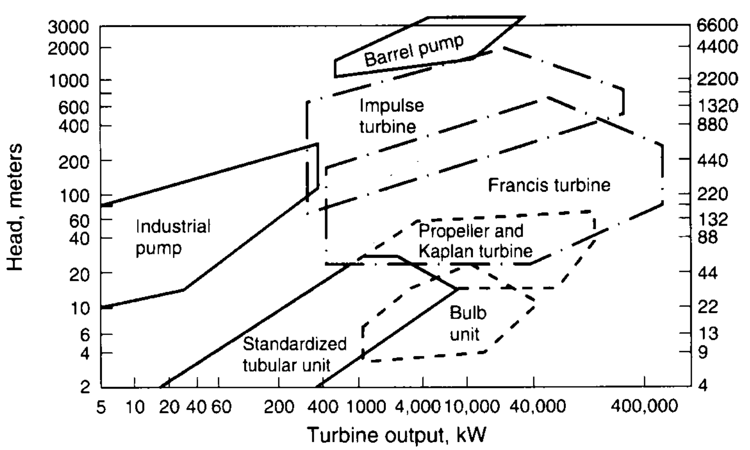


FIGURE 60. Traditional operating regimes of hydraulic turbines. New designs allow some turbines to cross traditional boundaries. (*Power.*)

TABLE 5. Performance Characteristics of Common Hydroturbines

	Operating head range		Capacity range	
Type	Rated head, ft (m)	% of rated head	MW	% of design capacity
Vertical fixed-blade propeller	7–120 (3–54) and over	55–125	0.25–15	30–115
Vertical Kaplan (adjustable blades and guide vanes)	7–66 (3–30) and over	45–150	1–15	10–115
Vertical Francis	25–300 (11–136) and over	50–150 and over	0.25–15	35–115
Horizontal Francis	25–500 (11–227) and over	50–125	0.25–10	35–115
Tubular (adjustable blades, fixed gates)	7–59 (3–27)	65–140	0.25–15	45–115
Tubular (fixed blades, adjustable gates)	7–120 (3–54)	55–150	0.25–15	35–115
Bulb	7–66 (3–30)	45–140	1–15	10–115
Rim generator	7–30 (3–14)	45–140	1–8	10–115
Right-angle-drive propeller	7–59 (3–27)	55–140	0.25–2	45–115
Cross flow	20–300 (9–136) and over	80–120	0.25–2	10–115

Source: Power.

To reduce capital cost, most site developers choose standard-design hydroturbines. With essentially every high-head site developed, low-head sites become more attractive to developers. Table 5 shows the typical performance characteristics of hydroturbines being used today. Where there is a region of overlap in Table 5 or Fig. 60, site-specific parameters dictate choice and whether to install large units or a greater number of small units.

Delivery time and ease of maintenance are other factors important in unit choice. Further, the combination of power-generation and irrigation services in some installations make hydroturbines more attractive from an environmental view because two objectives are obtained: (1) “clean” power, and (2) crop watering.

Maintenance considerations are paramount with any selection; each day of downtime is lost revenue for the plant owner. For example, bulb-type units for heads between 10 and 60 ft (3 and 18 m) have performance characteristics similar to those of Francis and tubular units, and are often 1 to 2 percent more efficient. Also, their compact and, in some cases, standard design makes for smaller installations and reduced structural costs, but they suffer from poor accessibility. Sometimes the savings arising from the unit’s compactness are offset by increased costs for the watertight requirements. Any leakage can cause severe damage to the machine.

To reduce the costs of hydroturbines, suppliers are using off-the-shelf equipment. One way this is done is to use centrifugal pumps operated in reverse and coupled to an induction motor. Although this is not a novel concept, pump manufacturers have documented the capability of many readily available commercial pumps to run as hydroturbines. The peak efficiency as a turbine is at least equivalent to the peak efficiency as a pump. These units can generate up to 1 MW of power. Pumps also benefit from a longer history of cost reductions in manufacturing, a wider range of commercial designs, faster delivery, and easier servicing—all of which add up to more rapid and inexpensive installations.

Though a reversed pump may begin generating power ahead of a turbine installation, it will not generate electricity more efficiently. Pumps operated in reverse are nominally 5 to 10 percent less efficient than a standard turbine for the same head and flow conditions. This is because pumps operate at fixed flow and head conditions; otherwise efficiency falls off rapidly. Thus, pumps do not follow the available water load as well unless multiple units are used.

With multiple units, the objective is to provide more than one operating point at sites with significant flow variations. Then the units can be sequenced to provide the maximum power output for any given flow rate. However, as the number of reverse pump units increases, equipment costs approach those for a standard turbine. Further, the complexity of the site increases with the number of reverse pump units, requiring more instrumentation and automation, especially if the site is isolated.

Energy-conversion-efficiency improvements are constantly being sought. In low-head applications, pumps may require specially designed draft tubes to minimize remaining energy after the water exits from the runner blades. Other improvements being sought for pumps are: (1) modifying the runner-blade profiles or using a turbine runner in a pump casing, (2) adding flow-control devices such as wicket gates to a standard pump design or stay vanes to adjust turbine output.

Many components of hydroturbines are being improved to reduce space requirements and civil costs, and to simplify design, operation, and maintenance. Cast parts used in older turbines have largely been replaced by fabricated components. Stainless steel is commonly recommended for guide vanes, runners, and draft-tube inlets because of better resistance to cavitation, erosion, and corrosion. In special cases, there are economic tradeoffs between using carbon steel with a suitable coating material and using stainless steel.

Some engineers are experimenting with plastics, but much more long-term experience is needed before most designers will feel comfortable with plastics. Further, stainless steel material costs are relatively low compared to labor costs. And stainless steel has proven most cost-effective for hydroturbine applications.

While hydro power does provide pollution-free energy, it can be subject to the vagaries of the weather and climatic conditions. Thus, at the time of this writing, some 30 hydroelectric stations in the northwestern part of the United States had to cut their electrical output because the combination of a severe drought and prolonged cold weather forced a reduction in water flow to the stations. Purchase of replacement power—usually from fossil-fuel-fired plants—may be necessary when such cutbacks occur. Thus, the choice of hydro power must be carefully considered before a final decision is made.

This procedure is based on the work of Jason Makansi, associate editor, *Power* magazine, and reported in that publication.

USE OF SOLAR-POWERED PUMPS IN IRRIGATION AND OTHER SERVICES

Devise a solar-powered alternative energy source for driving pumps for use in irrigation to handle 10,000 gal/min (37.9 m^3/min) at peak output with an input of 50 hp (37.3 kW). Show the elements of such a system and how they might be interconnected to provide useful output.

Calculation Procedure:

1. *Develop a suitable cycle for this application*

Figure 61 shows a typical design of a closed-cycle solar-energy powered system suitable for driving turbine-powered pumps. In this system a suitable refrigerant is chosen to provide the maximum heat absorption possible from the sun's rays. Water is pumped under pressure to the solar collector, where it is heated by the sun. The water then flows to a boiler where the heat in the water turns the liquid refrigerant into a gas. This gas is used to drive a Rankine-cycle turbine connected to an irrigation pump, Fig. 61.

The rate of gas release in such a closed system is a function of (*a*) the unit enthalpy of vaporization of the refrigerant chosen, (*b*) the temperature of the water leaving the solar collector, and (*c*) the efficiency of the boiler used to transfer heat from the water to the refrigerant. While there will be some heat loss in the piping and equipment in the system, this loss is generally considered negligible in a well-designed layout.

2. *Select, and size, the solar collector to use*

The usual solar collector chosen for systems such as this is a parabolic tracking-type unit. The preliminary required area for the collector is found by using the rule of thumb which states: For parabolic tracking-type solar collectors the required sun-exposure area is 0.55 sq.ft. per gal/min pumped (0.093 m^2 per 0.00379 m^3/min) at peak output of the pump and collector. Another way of stating this rule of thumb is: Required tracking parabolic solar collector area = 110 sq.ft. per hp delivered (13.7 m^2/kW delivered).

Thus, for a solar collector designed to deliver 10,000 gal/min (37.9 m^3/min) at peak output, the preliminary area chosen for this parabolic tracking solar collector will be, A_p = (10,000 gal/min)(0.55 sq.ft./gal/min) = 550 sq.ft. (511 m^2). Or, using the second rule of thumb, A_p = (110)(50) = 5500 sq.ft. (511 m^2).

Final choice of the collector area will be based on data supplied by the collector manufacturer, refrigerant choice, refrigerant properties, and the actual operating efficiency of the boiler chosen.

In this solar-powered pumping system, water is drawn from a sump basin and pumped to an irrigation canal where it is channeled to the fields. The 50-hp (37.3-kW) motor was

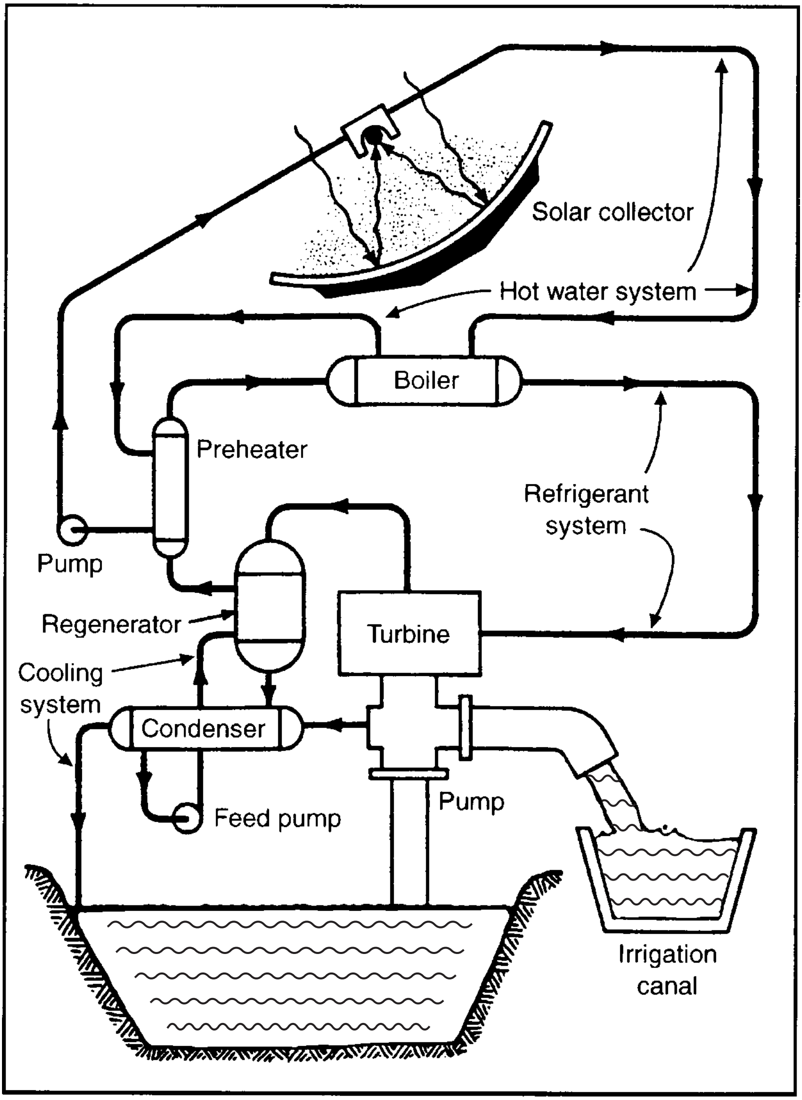


FIGURE 61. Closed-cycle system gassifies refrigerant in boiler to drive Rankine-cycle turbine for pumping water. (*Product Engineering,* Battelle Memorial Institute, and Northwestern Mutual Life Insurance Co.)

chosen because it is large enough to provide a meaningful demonstration of commercial size and it can be scaled up to 200 to 250 hp (149.2 to 186.5 kW) quickly and easily.

Sensors associated with the solar collector aim it at the sun in the morning, and, as the sun moves across the sky, track it throughout the day. These same sensing devices also rotate the collectors to a storage position at night and during storms. This is done to lessen the chance of damage to the reflective surfaces of the collectors. A backup control system is available for emergencies.

3. *Predict the probable operating mode of this system*

In June, during the longest day of the year, the system will deliver up to 5.6 million gallons (21,196 m^3) over a 9.5-h period. Future provisions for energy storage can be made, if needed.

Related Calculations. Solar-powered pumps can have numerous applications beyond irrigation. Such applications could include domestic water pumping and storage, ornamental fountain water pumping and recirculation, laundry wash water, etc. The whole key to successful solar power for pumps is selecting a suitable application. With the information presented in this procedure the designer can check the applicability and economic justification of proposed future designs.

In today's environmentally-conscious design world, the refrigerant must be carefully chosen so it is acceptable from both an ozone-depletion and from a thermodynamic standpoint. Banned refrigerants should not, of course, be used, even if attractive from a thermodynamic standpoint.

This procedure is the work of the editorial staff of *Product Engineering* magazine reporting on the work of Battelle Memorial Institute and the Northwestern Mutual Life Insurance Co. The installation described is located at MMLI's Gila River Ranch, southwest of Phoenix, Arizona. SI values were added by the handbook editor.